Interbank Credit Exposures and Financial Stability

Job Market Paper

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Abstract

This paper investigates how interbank credit exposures affect financial stability. Policy makers often see such exposures as undermining stability by exacerbating cascading losses through the financial system. I develop a model that features a trade-off between cascading losses and risk-sharing. In contrast to previous studies I find that reducing interbank connectivity may destabilize the financial system via the bank-run channel. This is because it decreases the risk-sharing benefits of interbank connectivity. A bank-run model features two islands that are connected via a long term debt claim. Varying the size of this claim (interbank connectivity), I study how the decision to 'run on the bank' is affected. I run a simulation of the model, calibrated to the U.S. banking system between 1997-2007. I find that large bankruptcy costs are required to trump the risk-sharing benefits of interbank credit exposures.

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I. Introduction

Should financial institutions be treated more favorably, in the context of a financial crisis, to other companies or individuals? This question has been the topic of active debate in recent years following the events of 2007-8. In practice, regulators the world over seem to answer it with a definite yes: from bail-outs to favorable treatment in bankruptcy, financial institutions are deemed systemically important and have earned an acronym to that effect (SIFI).[1]

Policy makers have argued that giving seniority to SIFIs could stabilize the financial system (Mengle, 2010). This is because it reduces interbank credit exposures, insulating the financial sector from cascading losses (domino-effect; Duffie and Skeel, 2012; Bliss and Kaufmann, 2004). However, giving seniority to banks implies that other creditors recover less in bankruptcy. Among those are short-term creditors whose panic may destabilize the financial system (Roe, 2013).[2] This paper investigates how interbank credit exposures affect financial stability. It studies the trade-off between domino-effect contagion and risk-sharing as it affects short-term creditors' decision to 'run on the bank'.

Studying this trade-off in equilibrium requires a model in which both the domino-effect and risk-sharing are present, and where non-bank creditors can respond to changes in the financial network. I propose a two-period bank-run model in which banks have two types of creditors: depositors and other banks. Depositors may decide whether to withdraw their deposit in period 1 or wait until long-term investment bears fruit in period 2. If enough of them withdraw in period 1, this decision to 'run on the bank' is individually optimal even though fundamentals could be high enough for it to be socially sub-optimal. Following Goldstein and Pauzner (2005, GP), I assume depositors don't have common knowledge about the economy's fundamentals; instead, each agent observes a private signal about fundamentals.[3] This allows me to evaluate existing policy in equilibrium, because it resolves the problem of multiple self-fulfilling equilibria typical of Diamond and Dybvig (1983) bank-run models.

[1] Systemically Important Financial Institutions.

[2] "If derivatives and repo counterparties bear less risk, as they do, due to the Bankruptcy Code's favoritism, then other creditors that are poorly prioritized bear more risk and thus have more incentive for market discipline."

[3] The noisy signal could also be which interpreted as agents' opinion about the soundness of the economy.

The first contribution of this paper is the finding that reducing interbank credit exposures may in fact decrease the stability of the financial system. In absence of bankruptcy costs, connecting the banks is always beneficial because it allows depositors to share idiosyncratic risk. This finding contrasts with previous studies on domino-effect contagion, who do not model agents' endogenous response to changes in the financial network (Acemoglu, Ozdaglar, and Tahbaz-Salehi, 2015; Gai and Kapadia, 2010).

The second contribution is to generate predictions as to which effect is stronger, depending on the magnitude of bankruptcy costs and degree of risk-aversion. If bankruptcy costs are positive, the risk-sharing channel conflicts with domino-effect contagion, as the latter reduces long-term value when the financial sector is highly interconnected. I simulate the model, calibrated to the U.S. banking sector between 1997-2007, and find that if bankruptcy costs are below 20%, the optimal policy is to have interbank connectivity in excess of 60% of banks' balance sheets. Conversely, if bankruptcy costs are as high as 50% netting is optimal. These results suggest that bank seniority could be counter-productive if bankruptcy costs are not too high.

The size of bankruptcy costs is important, because it determines the strength of the domino-effect. In the literature on domino-effect contagion, bankruptcy costs are often calibrated as high as 50-100%, referencing recovery rates on corporate bonds that of companies entered bankruptcy. However, it is problematic to ascribe low recovery rates to bankruptcy costs. The Lehman Brothers case serves as a good example. There, legal and administration costs amounted to about 2.5% of total claims approved by the courts (roughly $9 billion). Yet, general creditors recovered only 31% of their claims, which is significantly lower than the banking sector historical average.

This paper contributes to a number of areas in economics and finance. First, it is related to the literature on domino-effect contagion discussed above. My model is not the first that contains an endogenous response to instability. Erol and Vohra (2018) study endogenous network formation in face to the contagion risk. In their model, stability is a function of agents' choices, but is limited only to the domino-effect. The value of the network is at odds with its stability: the higher is the value of a link (financial or other) – the more agents want to connect – the larger is the impact of the domino-effect, thus decreasing stability. In contrast, the cause of instability in my paper is the interplay between exogenous and endogenous risk. Agents' choice of whether to run on the bank is directly affected by long term value: the higher is this value – the more stable is the banking sector. Interbank credit exposures enlarge this value because they allow agents to share idiosyncratic risk.

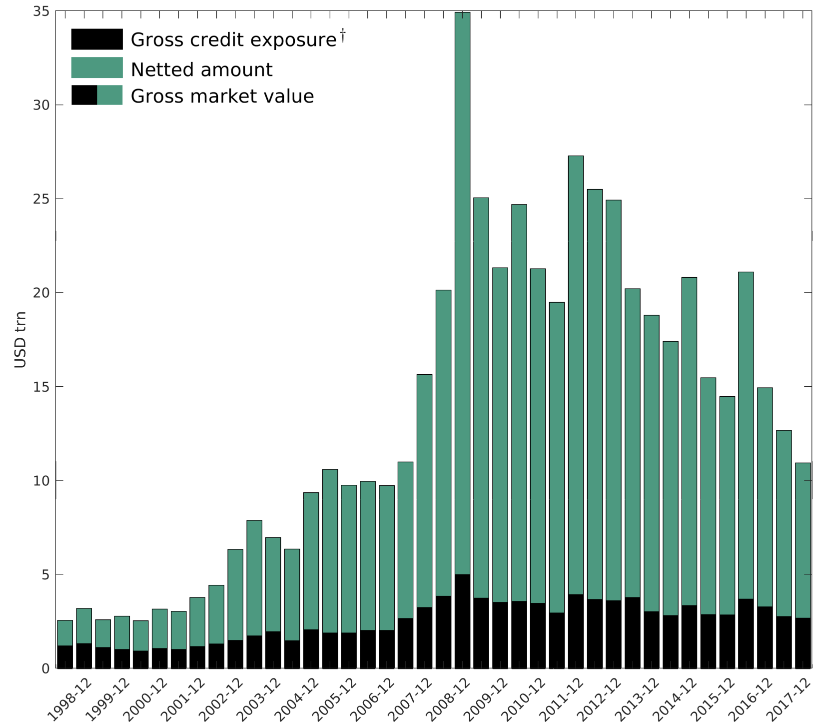
It also contributes to the area of banking by putting forth a novel mechanism by which risk-sharing increases financial stability. Other works in this vein modelled risk-sharing as occurring between banks themselves, e.g. via credit lines which smooth liquidity shocks (Allen and Gale, 2000; Ladley, 2013). The mechanism in my paper is different to theirs: interbank connectivity is only effective when banks are already insolvent; risk-sharing occurs at the level of depositors, not banks.

Another contribution to the banking literature is the application of the seminal work of GP to the study of public policy. Other works applied GP to study government guarantees (Allen, Carletti, Goldstein, and Leonello, 2018) and bank heterogeneity in banks' asset holdings (Goldstein, Kopytov, Shen, and Xiang, 2020). Both are applications using a similar framework but studying separate questions. Liu (2016) studies the joint occurrence of interbank credit freezes and bank-runs. His focus is on liquidity, while I study the interplay between liquidity and solvency.

In the literature on insolvency, my paper is related to Matta and Perotti (2016), who study how risk arising from premature liquidation of illiquid assets can contribute to the probability of a bank-run – and the way in which mandatory stay helps in dealing with this problem. In contrast, my paper focuses on priority rules that define the boundaries of the estate upon which mandatory stay is imposed. Bolton and Oehmke (2015) study how seniority rules affect banks' risk-taking behaviour. Their focus is the effect of seniority on investment efficiency and welfare, whereas mine is financial stability.

Seniority of interbank liabilities is achieved in practice by excepting Qualified Financial Contracts (QFCs, mostly derivative contracts) from mandatory stay. It has the effect of reducing interbank connectivity, as it allows SIFIs to net mutual assets and liabilities with an insolvent bank.[4] This would not

[4]The reason it is pertinent particularly to SIFIs is that it requires some form of mutual claims, which typically arise from their role as dealers in OTC derivative markets and as



† Gross credit exposure is the amount of credit risk left before collateralization and after netting; according to this number firms post initial margins.

Figure 1. Global OTC derivative markets (Data from the BIS)†

be allowed ordinarily. Figure 1 illustrates the trend in market value of OTC derivatives, which grew rapidly in the run-up to the 2007-8 crisis. In December 2008, netting reduced banks' credit risk by a staggering $30 trillion, about 85% of their global gross market value (Figure 1). Netting is one of the two most prominent exceptions to mandatory stay (Wood, 2007).

Insolvency netting was introduced to the U.S. Bankruptcy Code in a series of amendments between 1978-2006 (Mooney Jr, 2014).[5] This legislation was put forth "with congressional intent in creating... safe harbors to promote the stability and efficiency of financial markets." (Chapman, 2016) Other countries treat insolvency netting in different ways, deviating from the guidelines set up in the Basel Accords.[6] However, U.S. law is likely to be relevant in most

liquidity providers (Bliss and Kaufmann, 2004).

[5]In 1978, 'safe harbors' were first introduced to the Bankruptcy Code in order to enhance commodity market stability. Hence, a series of amendments passed by Congress expanded the type of institutions and contracts that could benefit from the protection of safe harbors, and consequently, netting became more and more robust. In the U.K., netting is an old legal practice, dating back as early as the 18$^{\text{th}}$ century, and is not limited to financial contracts.

[6]The Basel Accords set a minimum reporting requirement for two securities to be eligible

advanced economies, as banks may choose the legal system which governs their Master Netting Agreement.[7]

The paper is organized as follows. The next section sets up the model and section III describes the equilibrium. In section IV I explicitly analyze how an interbank connection affects the equilibrium and state the main result of the paper: giving priority to banks may increase the probability of runs. In section V I describe how I calibrate the model, and present simulation results. Section VI concludes with a discussion of the implications of this paper for policy. All proofs are provided in the the appendix.

II. Model

This section spells out a bank-run model with idiosyncratic risk based on Goldstein and Pauzner (2005, GP). In essence, bank-run models are a coordination game: there are situations in which if agents' believed that other agents will not run – they would not run themselves. The main problem these models deal with is the existence of a "bad" equilibrium (bank-run), which is Pareto-dominated – alongside a "good" equilibrium, which is Pareto-dominating. In the absence of uniqueness, bank-run models do not provide direction as to which equilibrium is selected, and thus policy analysis is precluded.[8]

GP apply methods from the theory on global games to Diamond and Dybvig's seminal bank-run model. By modelling the structure of the economy's fundamentals and agents' information, their model pins down a unique ex-ante probability of bank-runs. Moreover, one of their forceful points is to show that bank-runs are typically an equilibrium phenomenon even in the second-best case (equilibrium that is constrained by agents' private information about their type). In their model, bank-runs occur even in states of the world where

for set-off, that countries can only make more strict, i.e. less favourable to netting.

[7]From conversations I had with practitioners, they estimated that in the majority of cases the law governing set-off would be either American or English – mainly due to the advantages related to certainty attributed to common law – the difficulty of a judge to set a precedent under a common-law system, in contrast to the ability of a judge to interpret the law in her own way under a code-of-laws system.

[8]Multiplicity of equilibria in coordination games can be thought of as an artefact of extreme and implausible assumptions about common knowledge. These assumptions are intended to simplify analysis, i.e. they are not the result of an underlying reason internal to the logic of the model.

fundamentals are strong enough so that in absence of the coordination problem there would not be a run on the bank. In this sense bank-runs are a self-fulfilling prophecy.[9]

A. Physical Environment

An economy with two islands, a single consumption good and two assets (short and long) exists for three periods: 0,1 and 2. For the moment consider only an individual island; the islands are identical ex-ante. Island k is inhabited by a continuum of depositors and a representative bank. All agents have access to a risk free asset (with a return normalized to 0) and a risky asset that takes 2 period to mature; if allowed to mature, it yields θ_k. Total return on the risk-free and risky asset is given by $R(\theta_k)$,

$$R\ (\theta_k)\ =\ x + (1-x)\theta_k \tag{1}$$

where x is the investment in the short asset. Returns are a function of a state variable $\theta_k \in \Theta \subseteq \mathbb{R}$ (economic fundamentals); R is continuous and monotonically increasing in θ_k, so high values of the state variable imply good news to investors. The fundamentals in island k are composed of a common factor θ and a mean zero idiosyncratic factor ν_k,

$$\theta_k = \theta + \nu_k \tag{2}$$

$$\theta \sim f_\theta \quad \mathbb{E}[\theta_k] = \mu_\theta \quad (\nu_k, \nu_{-k}) \sim f_\nu \quad \mathbb{E}[\nu_k] = 0$$

Let $\boldsymbol{\theta} = (\theta, \nu_k, \nu_{-k})$ be the state of nature, and its probability density function $f_{\boldsymbol{\theta}}$. Since the islands are ex-ante identical, f_ν is symmetric: $f_\nu(\nu, \nu') = f_\nu(\nu', \nu)$, which also implies $Var(\nu_k) = Var(\nu_{-k})$. I assume that the correlation between

[9]The literature on self-fulfilling prophecies and coordination games dates back at least to Aumann (1976) who establishes the notion of common knowledge for game-theoretic models. Models with multiple equilibria were studied in banking (Chari and Jagannathan, 1988), monetary policy (Benhabib, Schmitt-Grohé, and Uribe, 2001) and macroeconomics (Farmer and Benhabib, 1994). Deviations of the common knowledge assumption gave rise to a vast literature that this review cannot hope to span. Notable references are Rubinstein (1989); Monderer and Samet (1989); Carlsson and Van Damme (1993); Morris and Shin (1998).

ν_k and ν_{-k} is not perfectly positive,

$$\frac{Cov(\nu_k, \nu_{-k})}{Var(\nu)} \neq 1 \tag{3}$$

B. *Agents and Preferences*

The economy is populated by a unit continuum of depositors, and one bank per island. There are two types of depositors: impatient (with proportion λ) and patient (with proportion $1 - \lambda$). Impatient depositors consume in period 1 and patient in period 2. Depositors don't know their type in period 0, which they observe only in period 1. Moreover, their type is their private information and is unverifiable, so that patient depositors can always disguise themselves as impatient. Ex-ante expected utility of an agent is given by

$$\mathbb{E}u = \int_{\boldsymbol{\theta}} \Big[n\ q\ u(c_1) + \big(1 - n + (n - \lambda)(1 - q)\big)\ u(c_2) \Big] f_{\boldsymbol{\theta}}(\boldsymbol{\theta}) d\boldsymbol{\theta} \tag{4}$$

where $\boldsymbol{\theta}$ is the state of nature, $n = n(\theta)$ is the proportion of depositors who withdraw in period 1, $c_t = c_t(\boldsymbol{\theta})$ is consumption in period t and $q = q(\theta)$ is the probability of consumption in period 1. In case an impatient depositor couldn't get her deposit, she consumes zero. If consumption in period 2 is positive, then the probability of consuming is 1; I therefore omit the probability of consumption in period 2 (and consequently its t subscript). The utility function u is a monotonically increasing ($u' > 0$) and has decreasing marginal gain ($u'' < 0$).

I assume that banks expect zero profits, thus maximizing agents' welfare. Banks face a portfolio allocation problem, investing x in the risk-free asset that yields R_f, and $1 - x$ in the risky asset with expected return of μ_θ. In all, they choose (c, x) in order to maximize depositors' period 0 expected utility (eq. 4).

C. *Information Structure*

Recall that returns on island k depend on a common factor θ and an idiosyncratic factor ν_k (eq. 2). The idiosyncratic factor (ν_k, ν_{-k}) is observed only in period 2. The common factor θ is observed imperfectly already in period 1. Agent i observes her type, and a private noisy signal θ_i

$$\theta_i = \theta + \varepsilon_i \tag{5}$$

$$\varepsilon \sim U[-\epsilon, \epsilon] \quad \varepsilon \perp \theta$$

where ε_i is the realization of the noise in agent i's signal; ε is uniformly distributed with range 2ϵ around the real value of θ. The rest of the details about agents preferences and the physical environment as well as the institutional environment, are all common knowledge. No information is revealed publicly, and for simplicity assume that there is no communication possible between agents.[10]

D. Institutional Environment

The rationale for a banking sector is risk-sharing of individual liquidity risk – i.e. being patient or impatient. In period 0, agents can deposit their funds at their regional bank, which promises a convertible debt contract with gross interest rate c if the deposit is withdrawn in period 1, and otherwise a debt-plus-equity claim on the assets of the bank in period 2.[11][12]

Denote by D the size of the period 2 debt claim[13] and by A_k the external assets of bank k from its own portfolio, (i.e. unrelated to bank $-k$)

$$A_k = (1 - \min[nc, x])R_k \tag{6}$$

[10] Even if it was possible to communicate, agents would not have had proper incentive to tell the truth about their signal, thus an unverifiable signal is enough.

[11] Observe that in absence of any other period 2 debt claims, the debt-plus-equity claim is equivalent to a simple equity claim. Demandable debt is a standard assumption in the banking literature for at least two reasons: (1) it is the optimal contract to overcome moral hazard (Calomiris and Kahn, 1991); (2) it is a constrained-optimal risk-sharing contract between agents in period 0 facing asymmetric information in period 1: "banks can be viewed as providing insurance that allows agents to consume when they need to most. Our simple model shows that asymmetric information lies at the root of liquidity demand." (Diamond and Dybvig, 1983)

[12] Convertible debt is a risk-sharing mechanism. It is not optimal ex-ante because it fixes the interest rate where market clearing would have it vary. However, it does addresses the asymmetric information problem, thus allowing agents to share liquidity risk (Diamond and Dybvig, 1983). If agents could coordinate on the good equilibrium (no run), then it is the constrained optimal contract.

[13] For the sake of brevity, I abstract from optimal capital structure.

In period 0 the bank invests x in the risk-free, and $1-x$ in the risky asset; period 1 value is capped at x. In period 1 it liquidates proportionally from its total portfolio to pay for first period withdrawals.[14] Impatient depositors always withdraw their deposit in period 1. Patient depositors may decide whether to withdraw in period 1 or 2.

Definition 1 (Liquidity)**.** *A bank is liquid if the total amount of period 1 claims is lower than liquid assets, $nc \leq x$; otherwise it is deemed illiquid. When a bank is liquid, the asset is not allowed to mature – i.e. it is liquidated yielding a scrap value of $(1-\gamma_1)A_k$ in period 2. Denote by $\tilde{\gamma}_1 = \tilde{\gamma}_1(n)$ the value of illiquidity costs depending on the level of withdrawals,*

$$\tilde{\gamma}_1(n) = \begin{cases} 0 & x \geq nc \\ \gamma_1 & x < nc \end{cases} \tag{7}$$

Moreover, when a bank is liquid period 1 claims are paid with certainty, whereas when it is illiquid period 1 claims are paid on first-come-first serve basis. The probability of consumption in period 1 $q = q(n)$ is

$$q(n) = \begin{cases} 1 & x \geq nc \\ \frac{x}{nc} & x < nc \end{cases} \tag{8}$$

Definition 2 (Solvency)**.** *Bank k is solvent if the total level of assets in period 2 exceeds the total level of debt claims, $A_k(1-\tilde{\gamma}) \geq D$; otherwise it is deemed insolvent, and all general creditors are paid pro-rata. If a bank is insolvent in period 2, its estate is liquidated yielding a scrap value of $(1-\gamma_2)(1-\tilde{\gamma}_1)A_k$. Denote by $\tilde{\gamma}_2 = \tilde{\gamma}_2(\boldsymbol{\theta})$ the value of illiquidity costs depending on the level of withdrawals,*

$$\tilde{\gamma}_2(\boldsymbol{\theta}) = \begin{cases} 0 & A_k(1-\tilde{\gamma}_1) \geq D \\ \gamma_2 & A_k(1-\tilde{\gamma}_1) < D \end{cases} \tag{9}$$

Illiquidity costs γ_1 and bankruptcy costs γ_2 affect period 2 value. The distinction between them is crucial, since illiquidity costs do not contribute to the domino effect. This is because they are not *caused* by the insolvency of

[14]This assumption ensures that when agents run on the bank, returns in period 2 are lower.

another bank.

III. Equilibrium

In this section I describe the equilibrium of the model. In order to focus on this paper's contribution, I begin the analysis of equilibrium for a given choice (x, c). In the next subsection I describe optimal portfolio allocation / deposit rates. The first best (FB) resource allocation can be achieved when there is no asymmetric information, and is given by:

$$\mathbb{E}_\theta \left[\frac{u'\left((1-\lambda c^{FB})\right)}{u'(c^{FB})} R(\theta) \right] = 1 \tag{10}$$

This allocation is only available if agents' types are not private information; otherwise bank-runs (patient depositors withdrawing early) may disrupt this optimality. Bank-runs are a result of a failure of patient agents to coordinate on the 'good equilibrium'.

Definition 3 (Bank-runs)**.** *A 'good equilibrium' obtains if all patient depositors withdraw late whenever fundamentals are high enough to support the optimality of withdrawing late. Bank-runs occur in states in which, although fundamentals are high enough to support the good equilibrium, nevertheless there is at least some early withdrawal by patient depositors and a Pareto sub-optimal equilibrium obtains.*

Impatient agents always withdraw their funds in period 1. However, patient depositors need to choose whether to withdraw in period 1 or wait until period 2. Their decision is based upon all available information. Crucially, agents use their signal in two ways: (a) to infer information about exogenous variables: returns R_k; (b) to infer information about endogenous variables: the proportion of agents who withdraw early, n.

Let $v = v(\boldsymbol{\theta})$ be the patient agent differential utility between withdrawing late and early at state $\boldsymbol{\theta}$. The relevant welfare evaluation made by a patient agent is captured by $\Delta = \mathbb{E}[v\,|\,\theta_i]$, the expected utility difference between withdrawing late to withdrawing early.

Definition 4 (Patient agents' utility differential)**.** *Let $\Delta(\cdot)$ be patient agents' utility differential between withdrawing early to withdrawing late:*

$$\Delta(\theta_i) = \iiint_{\boldsymbol{\theta}} v(\boldsymbol{\theta}) f_{\boldsymbol{\theta}}(\boldsymbol{\theta}|\theta_i) d\nu_k d\nu_{-k} d\theta \tag{11}$$

$$\begin{aligned} v(\boldsymbol{\theta}) &= u(c_2) - [qu(c_1) + (1-q)u(c_2)] \\ &= q[u(c_2) - u(c_1)] \end{aligned} \tag{12}$$

where $q = q(\theta) = \min[\frac{x}{nc}, 1]$ (see eq. 8)

$\Delta(\theta_i)$ indicates the expected value of v, conditional on observing signal θ_i. v depends on the proportion of withdrawals n, the short-term interest rate c, liquid assets x, returns R and the capital structure in period 2. In keeping with the global games method, I assume at least some values of fundamentals in which patient agents' binary decision (withdraw early or late) does not depend on other agents' actions:

Assumption 1 (Extreme locales of fundamentals)**.** *Two extreme locales of fundamentals are assumed to exist in which patient agents' actions are independent of their beliefs concerning other agents' actions.*

a. Lower dominance locale: $\theta \in [0, \underline{\theta}(c, x)]$, where $\underline{\theta}$ is defined by the relation

$$c = \frac{1-\lambda c}{1-\lambda} R(\underline{\theta}) \tag{13}$$

b. Upper dominance locale: $\theta \in [\bar{\theta}(c, x), 1]$, where

$$\frac{(1-x)R(\bar{\theta})(1-\gamma_1)(1-\gamma_2)}{1-\lambda} \geq c \tag{14}$$

Furthermore, $\bar{\theta}$ is assumed to satisfy $\bar{\theta} < 1 - 2\epsilon$.

The interpretation of assumption 1 is as follows: a. if fundamentals are low enough so that the consumption in period 1 is equated to that of period 2 even in the best of states (no run) – then a patient depositor's strictly dominant strategy is to withdraw early regardless of other agents' actions; b. if fundamentals are high enough so that consumption in period 2 is higher than in period 1 even in the worst of states – then a patient depositor's strategy is to withdraw late regardless of other agents' actions.

Let $s(\theta_i)$ be patient depositor i's (mixed) strategy – the probability with which she withdraws early. It is equal to 1 (0) whenever $\Delta(\theta_i, \cdot) < 0$ (> 0). If in equilibrium s^* is increasing from 0 to 1 at some θ^* and is 1 for $\theta_i < \theta^*$ and 0 for $\theta_i > \theta^*$, I call this a *threshold equilibrium* (θ^* is a threshold value of the signal below which she withdraws early even if she is patient).

Proposition 1 (Unique threshold-equilibrium)**.** *There exists a unique equilibrium. Patient agents withdraw early whenever they observe a signal $\theta_i < \theta^*$, and withdraw late otherwise.*

All proofs are provided in the the appendix. In a threshold equilibrium the proportion of depositors withdrawing in period 1 in each island n:

$$n(\theta, \theta^*) = \begin{cases} \lambda & \text{if } \theta \in [\theta^* + \epsilon, 1] \\ \lambda + (1-\lambda)\left(\frac{1}{2} + \frac{\theta^* - \theta}{2\epsilon}\right) & \text{if } \theta \in [\theta^* - \epsilon, \theta^* + \epsilon] \\ 1 & \text{if } \theta \in [0, \theta^* - \epsilon] \end{cases} \tag{15}$$

Note that since $\varepsilon_i \sim U(-\epsilon, \epsilon)$, then all signals are within the bounds of $\pm\epsilon$ from the common factor θ, and in a given threshold-equilibrium (θ, θ^*): $n(\theta, \theta^*)$ is degenerate. For values of $\theta \geq \theta^* + \epsilon$ no patient agent will withdraw early, while for $\theta \leq \theta^* - \epsilon$ all patient agents withdraw early. In between these two values

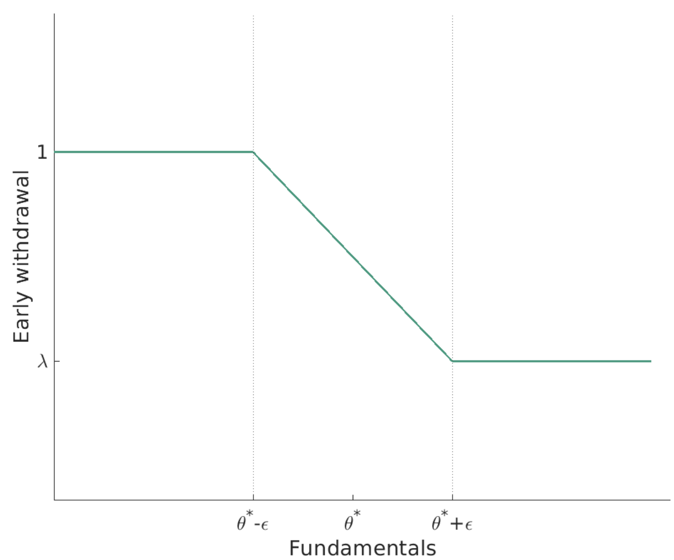


Figure 2. Proportion of early withdrawals $n(\theta)$

the proportion of depositors withdrawing early is linearly decreasing in θ with slope $\frac{1-\lambda}{2\epsilon}$ (see Figure 2).[15]

There are three main differences between my model and that of GP: an additional island, asset returns and bankruptcy costs. First, in GP's model there is a single island, which precludes any heterogeneity in period 2. Second, GP have a fixed return with varying probability, while in my model returns vary with θ_k, but are certain for a given θ_k. Third, my model has bankruptcy/illiquidity costs whereas GP's model doesn't. In both models there are no costs to liquidating the long asset up to a proportion x, at which point first period claims are paid on a first-come-first-served basis. However, in GP $x = 1$ so there are no costs at all. Conversely, in my model liquidation of the long asset is cost-free up to a proportion $x \leq 1$. Not all assets are liquidated to satisfy first period withdrawals, which means there is always some value left to agents who did not run or didn't manage to withdraw their deposit in time.

This concludes the setup of the model. In sum, I spelled out a variation of an otherwise standard bank-run model, originally put forward by Diamond and Dybvig (1983). Crises are a result of depositors inability to coordinate on the good equilibrium. Equilibrium multiplicity is resolved by relaxing the assumption of common knowledge about economic fundamentals (and therefore other agents' behaviour). Depositors observe noisy signals about fundamentals, and use a threshold strategy in equilibrium – as in Goldstein and Pauzner (GP).

A. Optimal Portfolio Allocation and Deposit Rate

The analysis above holds for any choice of (x, c). Banks choose (x, c) to maximize agents' ex-ante utility. Since the model has no closed form solutions for θ^* as a function (x, c), it is impossible to state an exact expression for x and c.

If markets were complete (no asymmetric information), the optimal risk-sharing scheme would imply a non-constant deposit rate. With asymmetric information, the choice of c is constrained to be constant because agents cannot make credible promises to pay.[16] In this case banks may choose $c \in \mathbb{R}_+$,

[15] Linearity is due to uniform distribution of the signal.

[16] If there were equity markets, the deposit rate would remain constant, but consumption in period 1 would not.

weighing two opposing effects. A higher deposit rate: (1) increases ex-ante welfare due to risk sharing; (2) decreases ex-ante welfare due to lower investment and higher threshold strategy θ^*. The first effect was first studied in (Diamond and Dybvig, 1983); the second was studied in GP.

Proposition 2 (Optimal Deposit Rate)**.** *For a given x, there exists a unique choice of deposit rate c which maximizes agents ex-ante utility.*

Proposition 3 (Optimal Liquidity)**.** *For a given c, there exists a unique choice of liquid assets x which maximizes agents ex-ante utility. If illiquidity costs are low, $\gamma_1 \leq \bar{\gamma}_1$, it is optimal to choose $x = \lambda c$ (the minimum required to pay impatient agents' withdrawals); in case $\gamma_1 > \bar{\gamma}_1$, banks choose $x(\gamma_1) > \lambda c$.*

Proposition 3 states the trade-off that banks face when choosing liquidity. On one hand, a more liquid portfolio of assets reduces returns. On the other hand, it salvages value because the bank is liquid in more states. For these reasons, higher liquidity may be beneficial for stability, but not always. The optimal choice of x equates the marginal effect of liquidity on period 2 utility (negative) to the expected value salvaged due to higher liquidity (positive).

IV. Interbank Connectivity

Section II provided the general setup of the model, with the aim of giving rise to a framework in which banks have two types creditors: depositors and banks. In the last section I described the equilibrium. In this section I consider an interconnected financial system: banks who have mutual assets and liabilities.

Banks have a debt claim of size B on one another, which matures in period 2. Recall that the size of non-bank liabilities is D, and denote by $\psi = \frac{B}{B+D}$ banks' recovery rate. If at least one bank is insolvent, the interbank connection implies a transfer from the strong to the weak bank. Let $\tilde{A}_k$ be the value of bank k's external assets in period 2 including bankruptcy costs

$$\tilde{A}_k = A_k \times \Gamma \tag{16}$$

$$\Gamma = \overbrace{(1 - \tilde{\gamma}_1)}^{\text{Illiquidity costs}} \times \overbrace{(1 - \tilde{\gamma}_2)}^{\text{Bankruptcy costs}}$$

where the definition of $\tilde{\gamma}_2$ is altered in order to reflect the interbank connection

$$\tilde{\gamma}_2(\boldsymbol{\theta}) = \begin{cases} 0 & A_k(1-\tilde{\gamma}_1) + \min[\psi(\tilde{A}_{-k}+B), B] \geq D + B \\ \gamma_2 & (1-\tilde{\gamma}_1) + \min[\psi(\tilde{A}_{-k}+B), B] < D + B \end{cases} \tag{17}$$

Connecting the banks with mutual assets and liabilities directly alters consumption in period 2 in two ways: on the one hand it effects a payment from the island with high fundamentals to the one with low fundamentals if at least one of them is bankrupt (risk sharing); on the other hand it transmits bankruptcy/illiquidity costs between islands (domino effect contagion).[17]

There are four possible cases, which I call regions, depending on which bank is solvent or not: (region 1) both banks solvent; (region 2) k solvent, $-k$ insolvent; (region 3) k insolvent, $-k$ solvent; (region 4) both banks insolvent. Table I summarizes all possibilities, and provides the conditions for being in each region depending on (A_k, A_{-k}).

Equations 18-21 state consumption in period 2 in island k for each possibility. If both banks are solvent (region 1), consumption is the same as it would have been in absence of interbank connection. Consumption in island k depends only the fundamentals in of bank k,

$$c_2(\boldsymbol{\theta}|\boldsymbol{\theta} \in \boldsymbol{\Theta}_1) = \frac{\tilde{A}_k}{1-\min[n, x/c]} \tag{18}$$

In case bank k is solvent and bank $-k$ is not (region 2), consumption in island k depends on the level of fundamentals for both banks,

$$c_2(\boldsymbol{\theta}|\boldsymbol{\theta} \in \boldsymbol{\Theta}_2) = \frac{\tilde{A}_k - B + \psi[B + \tilde{A}_{-k}]}{1-\min[n, x/c]} \tag{19}$$

If bank $-k$ is solvent but bank k is not (region 3), consumption in island k is higher than it would absent of an interbank connection, but does not depend on the fundamentals of $-k$. Depositors in island k consume a proportion $1-\psi$

[17] By setting agents noisy signals around the common factor, I preclude island heterogeneity in period 1, thus illiquidity costs will play no role.

of total assets available to them,

$$c_2(\boldsymbol{\theta}|\boldsymbol{\theta} \in \boldsymbol{\Theta}_3) = \frac{(1-\psi)(\tilde{A}_k + B)}{1 - \min[n, x/c]} \tag{20}$$

Finally, when both banks are insolvent (region 4), depositors in island k consume a proportion $\frac{1}{\psi}$ from bank k's external assets, and $\frac{\psi}{1+\psi}$ from bank $-k$'s external assets,

$$c_2(\boldsymbol{\theta}|\boldsymbol{\theta} \in \boldsymbol{\Theta}_4) = \frac{\frac{1}{1+\psi}\tilde{A}_k + \frac{\psi}{1+\psi}\tilde{A}_{-k}}{1 - \min[n, x/c]} \tag{21}$$

Table I. State-space partition

	Solvency		
Region	Bank k	Bank $-k$	Condition
1	Yes	Yes	$A_j(1-\tilde{\gamma}_1) \geq D \ \forall j \in \{k, -k\}$
2	Yes	No	$A_{-k}(1-\tilde{\gamma}_1) < D \ \wedge$ $A_k(1-\tilde{\gamma}_1) \geq D + B(1-\psi) - \psi\tilde{A}_{-k}$
3	No	Yes	$A_k(1-\tilde{\gamma}_1) < D \ \wedge$ $\tilde{A}_{-k} \geq D + B(1-\psi) - \psi\tilde{A}_k$
4a	No	No	$A_j(1-\tilde{\gamma}_1) < D \ \forall j \in \{k, -k\}$
4b	No*	No	$A_{-k}(1-\tilde{\gamma}_1) < D \ \wedge$ $D \leq A_k(1-\tilde{\gamma}_1) < D + B(1-\psi) - \psi\tilde{A}_{-k}$
4c	No	No*	$A_k(1-\tilde{\gamma}_1) < D \ \wedge$ $D \leq \tilde{A}_{-k} < D + B(1-\psi) - \psi\tilde{A}_k$

*But would have been absent interbank connection

Connecting the banks changes expected utility from consuming in period 2. As before, utility in island k is (strictly) increasing in the fundamentals of bank k, though this time it is also (weakly) increasing in the fundamentals of bank $-k$. Bank interconnectivity may affect the probability of consumption in period 1 only via θ^*, i.e. through agent endogenous choice of whether to run on the bank. Otherwise, it does not affect period 1 value.

Corollary 1. *There exists a unique threshold equilibrium even if banks are interconnected.*

What effect might varying B have on the incentive of patient agents to run on the bank? On the one hand, we can see that bank inter-connectivity has an insurance effect. To the extent that agents are risk averse, they would prefer a more equal consumption across islands. On the other hand, this insurance may come at a cost. By pushing both banks over the cliff-edge of bankruptcy costs instead of just one, connecting the islands may destroy value in some states (the domino-effect).

Proposition 4 (Insolvency netting)**.** *If* $\gamma_2 = 0$ *(no bankruptcy costs due to insolvency), optimal interbank connectivity implies merging the banks (*$B \to \infty$*). If* $\gamma_2 > 0$ *the effect of interbank connectivity on stability is ambiguous*

The reason interbank connectivity unambiguously decreases the probability of bank-runs if $\gamma_2 = 0$, is that bankruptcy costs are incurred only via period 1 illiquidity. The domino effect is absent due to the assumption that agents observe a noisy signal of the *common factor* θ. In this case connecting the banks can only increase expected welfare in period 2 due to the insurance effect. If $\gamma_2 > 0$ there is a trade-off between insurance and bankruptcy costs. Insolvency netting has the effect of reducing interbank connectivity. In no case does insolvency netting increase financial stability unambiguously, as is suggested by the networks literature.

V. Comparative Statics

In the previous Section I demonstrated how higher interbank connectivity might enhance the stability of the financial sector via the bank-run channel. Proposition 4 shows that in absence of bankruptcy costs in period 2 ($\gamma_2 = 0$), interbank connectivity is always beneficial to stability due to its insurance effect. If, however, $\gamma_2 > 0$ then interbank connectivity may also undermine stability due to the domino effect. The prediction of the model is thus ambiguous. In this section I present simulation results for a set of calibrated parameters, with the aim of resolving this ambiguity in prediction.

A. Calibration: Preferences, Technology and Information

I make assumptions about the utility, technology and the distribution of θ and ν_k. The utility function exhibits constant relative risk aversion:

$$u(c_t) = 1 - \exp(-\alpha \; c_t)$$

where the CARA parameter α ranges between 0.1 to 4 (benchmark case: $\alpha = 1$). The values of λ are chosen so to be consistent with observed levels of deposit rates and liquid asset holdings, $x > \lambda c$. Returns on total assets are linear in θ:

$$R_k = R\ (\theta_k, x) = xR_f + (1-x)\theta_k \qquad \theta_k = \theta + \nu_k$$

where R_f is the gross risk free rate and θ_k the return on the long asset. I calibrate R_f using 3-months T-Bill rates which averaged 3.62% p.a. between 1997-2007. The common component in returns independent from the idiosyncratic components, who are also themselves independent and distributed normally.

$$\begin{pmatrix} \nu_k \\ \nu_{-k} \end{pmatrix} = N\left(\mathbf{0},\ \sigma_\nu^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$$

I calibrate σ_ν using the volatility of 3-months cumulative returns on the S&P 500 (roughly 7.4%, or 16% annually for iid processes). Deviation from common knowledge (ϵ) is matched to half the interquantile range of SPF[18] forecasts, 1.5% per annum, or about 0.37% over three months.

[18]Survey of Professional Forecasters, see further details below.

Table II. Calibration: Preferences, Technology and Information

Parameter Reference	Parameter	Value(s)	Data Reference	Value(s)
CARA	α	1.0		
Impatient depositors	λ	0.05		
Risk free rate	R_f	1.018	3-months TBill	0.88%
Return S.D. (long asset)*	σ_ν	0.074	S.D. of S&P 500 3-months cumulative return	7.4%
Signal noise	ϵ	0.0037	SPF interquantile range	0.74%

*Idiosyncratic component

B. *Calibration: Banking Sector*

Short-term deposit rates are calibrated based on financial-sector commercial paper rates with maturity 1-3 months (source: Federal Reserve H.15 Selected Interest Rates).[19] Liquid assets x is calibrated based on the ratio of high quality liquid assets (HQLA) to total assets. For a sample of large U.S. bank holding companies in mid-2007, values ranged between 3% to 9% (Yankov et al., 2020). Non-bank debt claims D are calibrated to match the leverage ratio for a sample of the largest U.S. banks between 1997-2007: Leverage ratio $= 1/(1-D) = 25$.[20] Realistic values for interbank mutual claims can be gauged via data on derivative assets and liabilities of U.S. bank holding companies from the Fed (Annual Report of Holding Companies - FR Y-6). Gross market values vary between 30%-150% of banks' balance sheets.[21]

Finally, we need to input values for illiquidity and bankruptcy costs (γ_1

[19] In order to match the model frequency, the annual figure needs adjustment to a 3-months frequency, $(1 + \text{annualized rate})^{1/4}$.

[20] A higher value of D strengthens the domino effect, since it implies that bankruptcy costs are borne more often.

[21] Note that derivative trading assets are presented by large banks as an off-balance sheet item, which is why it can be higher than the balance sheet.

and γ_2 respectively). Since in my model γ_1 doesn't contribute to the domino effect, I shall set it to zero. All costs will be incurred via γ_2, so that the domino effect may have it's largest effect. γ_2 determines whether interbank connectivity enhances stability or undermines it, since it controls the magnitude of the domino effect. Due to the sensitivity of the results to this parameter, I use it in comparative statics rather than attempt to provide a point estimate.

In order to map the bankruptcy costs parameter γ_2 into the the economic magnitude of costs in practice, one also has to take account of the probability of default. The unconditional probability of default is not matched well by the model: if the unconditional expectation of quarterly returns is 2.4% and the standard deviation 7.4%, a leverage ratio of 25 imply that banks in the model default around 3% of the time. In reality, investment grade corporate bonds default about 0.5% of the time. This then means γ_2 has an increased impact compared to its real-world counterpart, implying a conservative approach to estimating optimal interbank exposures.

Table III. Calibration: Banking Sector

Parameter Reference	Parameter	Value(s)	Data Reference	Value(s)
Short-term bank rate (short asset; net)	c	1.01	Commercial paper – financials (3-months)	0.96%
Liquid assets	x	0.06	HQLA	3% - 9%
Non-bank debt claims	D	0.96	Leverage ratio	25
Interbank mutual claims	B	0 - 1.5	Gross market value of derivatives	30-150%
Illiquidity costs	γ_1	0		
Bankruptcy costs	γ_2	0-0.015		

C. *Calibration: Target*

The main output of the model is the threshold level of expected returns at which bank-runs occur. This is also the variable my calibration aims to match.[22] It is important to establish a benchmark probability of bank-runs that the model would target, i.e. a level of θ^* that would be in accord with historical experience.

Due to the rarity of bank-runs, I refrain from matching a hard target. Rather, I will match a wide area below expected returns, using the median forecast of average 10-year returns on the S&P 500 Index (available from the Survey of Professional Forecasters, SPF).[23] This proxies for investors' expec-

[22]In order to deduce from this output a frequency at which bank-runs occur, one would have to make a stand on the unconditional distribution of agents' expectations.

[23]The Federal Reserve Bank of Philadelphia surveys a panel of professional forecasters on variables of interest on a quarterly basis. Since 1992-Q1, they began collecting forecasts of average 10-year returns on the S&P 500, collected on an annual basis. I use the median forecast 2008-Q1 as a target θ^* for the model to match.

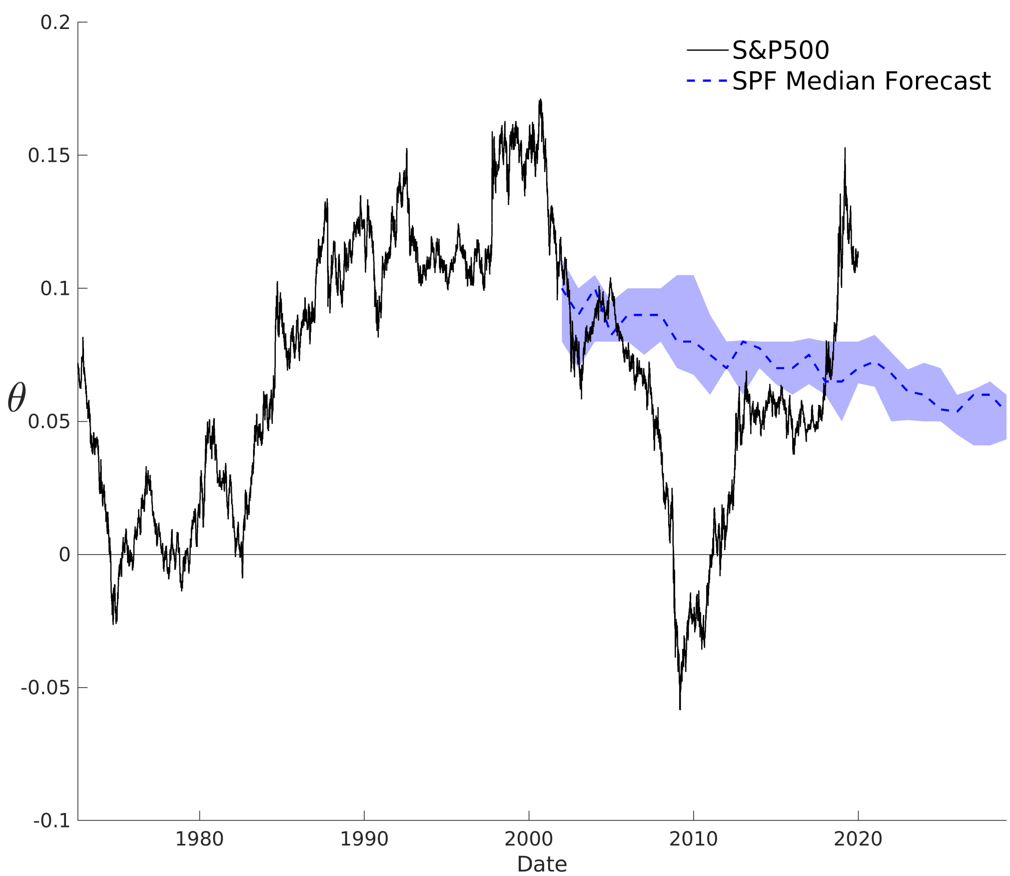


Figure 3. S&P 500 10-year Average Returns and SPF Forecasts

tations about the value of long term investment. Figure 3 plots the realized 10-year average returns on the S&P 500 between 1972-2020, along with the SPF median forecast (shifted by 10 years to match realized returns). Panelists' expectations are much less volatile than realized returns.[24] Between 1992-2020, they dropped by 450 b.p. from 10% to 5.5% per annum. I define a benchmark case where the time between periods is 3 months, thus I target threshold strategies that are lower than $(1+.1)^{1/4}-1 \approx 2.5\%$.

D. Simulation

In this subsection I outline my simulation methodology. I rely on numerical techniques to get calibration results. Since the model has no closed form solutions, it requires a fixed-point algorithm to locate θ^*, the threshold signal below which patient agents run on the bank. Locating this threshold strategy involves evaluating patient agents' expected differential utility Δ (equation 11).

The model is set up under minimal assumptions about utility and technology. In principle one could work with any odd function for these objects, so long as it satisfies generic conditions described in section II. Simply evaluate the triple integral for a candidate equilibrium θ_0^*,

$$\int_{\theta=\theta_0^*-\epsilon}^{\theta_0^*+\epsilon} \int_{\theta_k} \int_{\theta_{-k}} v(\cdot) f(\theta_k, \theta_{-k}|\theta) d\theta_k d\theta_{-k} d\theta$$

insert the result to the fixed point algorithm that will suggest the next candidate θ_1^*; then repeat. The process is concluded after T steps when $\Delta(\theta_T^*) \approx 0$.

It could be computationally costly to evaluate Δ in this way, especially due to multiple uncertainty sources (idiosyncratic risk plus uncertainty about the common component). The combination of CARA utility and Normal Distribution addresses this issue, because it yields partially closed form solutions.[25] Further details are provided in the technical appendix.

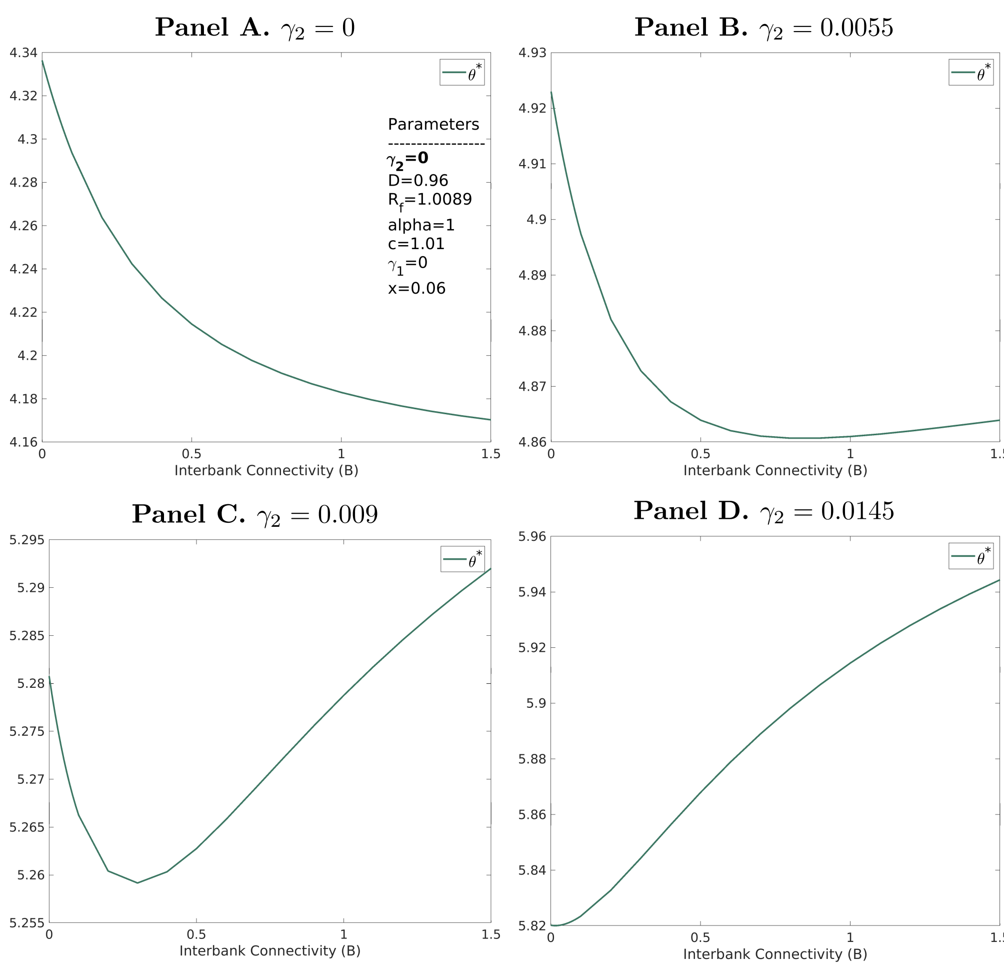
[24]Because of this I opt to match survey data. Asset prices are too volatile to be a good estimate of investors' long-term growth, and would result in an difficulty to generate a realistic threshold strategy.

[25]Essentially reducing the triple integral to components of a double / single integrals. This speeds up each evaluation from about 18 seconds to 1 second. Figure 8 demonstrates that the two algorithms' outputs are approximately the same.

E. Results

Figure 4 graphs the equilibrium threshold strategy θ^* as it varies with interbank connectivity (B) when bankruptcy costs are: (a) zero; (b) small (0.55% for 6 months, $\gamma_2 = 0.0055$ – 10% over 10 years); (c) intermediate (0.9% for 6 month, $\gamma_2 = 0.009$ – 17% over 10 years); (d) large (1.45% for 6 month, $\gamma_2 = 0.0145$ – 26% over 10 years). In line with Proposition 4, in absence of bankruptcy costs (Panel A), agents run on the bank less often (θ^* lower) when the banks are more highly interconnected.

For example, when the banks are not connected, agents run on the bank when they expect long-term returns to be 4.34% per annum; when mutual



θ^* is presented in annualized percent

Figure 4. Threshold strategy (θ^*), varying interbank connectivity (B); $\gamma_1 = 0$

claims are 150% of the balance sheet size, this number drops by 17 b.p. to 4.17%. Compare this to intermediate bankruptcy costs in Panel C. When interbank connectivity is 30% of the balance sheet size, agents run on the bank when they expect long-term returns to be 5.26% per annum; when mutual claims are 150%, this number increases by 3 b.p. to 5.29%. In this case the optimal policy is for an intermediate value of interbank connectivity.

We can see that bankruptcy costs have two effects on the probability of bank-runs. First, all else equal, higher bankruptcy costs mean agents run on the bank more often. This makes sense because if agents incur higher bankruptcy costs, a higher reward is necessary to dissuade them from running on the bank. Second, bankruptcy costs strengthen the domino effect, so that higher interbank connectivity is less beneficial. The former increases the level of the curves in Figure fig. 4; the latter shifts their minima to the left.

In Figure 5, I graph the level of optimal interbank connectivity, varying risk aversion and bankruptcy costs. Optimal interbank connectivity varies between 0-150% of banks' balance sheets. On the one hand, the higher is risk

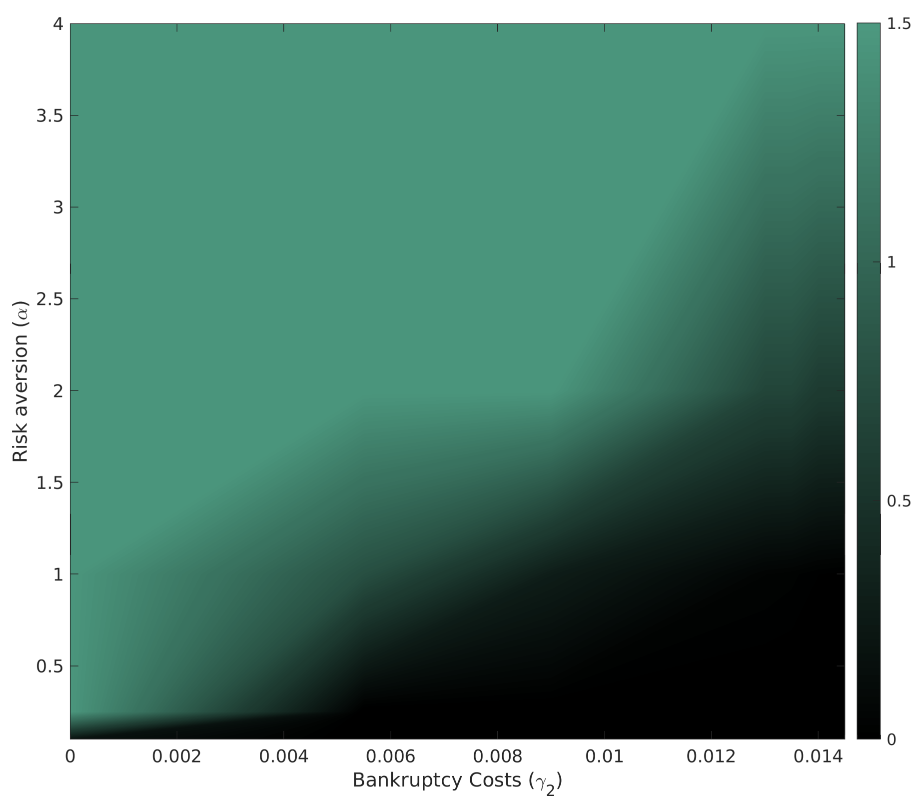
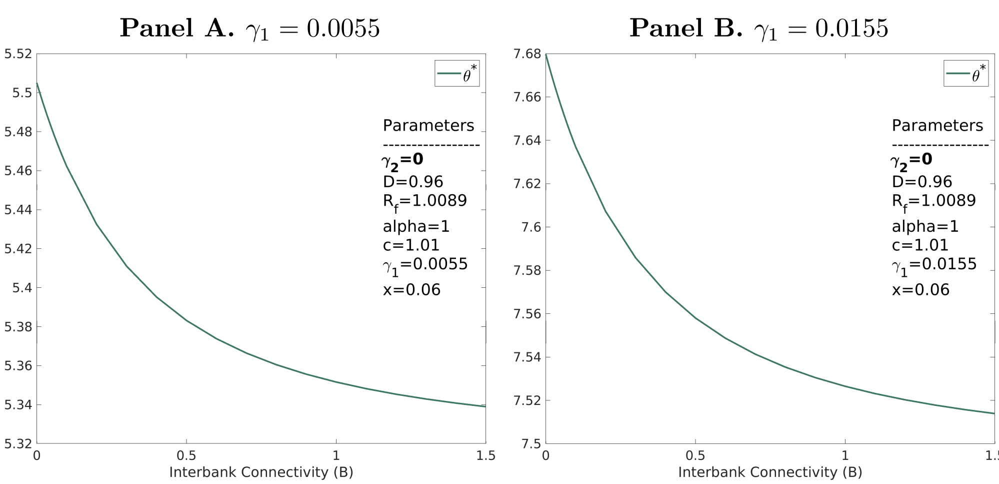


Figure 5. Optimal Interbank Connectivity

aversion the higher is the optimal interbank connectivity. This makes sense because higher risk aversion implies high risk sharing benefits from interbank connectivity. On the other hand, higher bankruptcy costs imply a lower optimal interbank connectivity. Higher costs due to bankruptcy imply a stronger domino-effect. In many of the cases, netting is not the optimal policy.

It is important to establish to economic magnitude of these effects on financial stability. A change of 3-17 b.p. in θ^* may seem small at first glance, especially considering the high volatility of equity markets. However, although it is true that long-term growth varies widely, the expectation of it varies much less (see Figure 3). The magnitude of changes should be compared to the latter. The interquantile range of forecasts of long term growth is 300 basis points.

Overall the model does a good job at matching a reasonable level of the threshold strategy. Values of θ^* ranging between 4.2%-6% p.a. (when $\gamma_1 = 0$) imply a spread between the threshold strategy and the deposit rate of 30-300 b.p., depending on the level of γ_2 and risk aversion. The size of this spread can be increased by assuming higher values of illiquidity costs. For example, assuming illiquidity costs of $\gamma_1 = 0.0155$ and no bankruptcy costs $\gamma_2 = 0$ implies roughly 300 b.p. increase in θ^* (see Figure 6). The effect of connecting the banks agrees with the model's predictions: in absence of bankruptcy costs, the optimal policy is to completely merge the banks – so as to achieve maximum risk sharing. The optimal level of interbank exposures decreases as we increase



θ^* is presented in annualized percent

Figure 6. Threshold strategy (θ^*), varying interbank connectivity (B); $\gamma_2 = 0$

bankruptcy costs. Illiquidity costs γ_1 have no impact on the domino-effect because banks are only heterogeneous in period 2; γ_1 would contribute toward the domino-effect if banks were heterogeneous in period 1.

VI. Discussion

This paper analyzes the effects of interbank connectivity on financial stability, with reference to a concrete policy – insolvency netting. Netting reduces the size of interbank connectivity by allowing banks to settle mutual assets/liabilities on a net basis. The rationale for this is to stabilize the financial system by preventing the spillover of bankruptcy costs in the event of a crisis (domino-effect contagion). However, this is done by concentrating the losses at a particular group of creditors, undoing the risk-sharing effect that an interconnected financial system provides.

To study the effect of interbank credit exposures in equilibrium, I propose a two period bank-run model with bank heterogeneity in period 2. Bank inter-connectivity has two opposing effects on stability: a domino-effect and an risk-sharing effect. In absence of bankruptcy costs ($\gamma_2 = 0$), I find that insolvency netting *decreases stability*; in case there are costs also due to insolvency ($\gamma_2 > 0$), the effect of netting is ambiguous.

These results stand in stark contrast to previous research on contagion in financial markets. Most of the work in this field neglect to take account of the endogenous response of rational agents to the limiting of financial interconnectivity. The welfare effect from limiting inter-connectivity has no impact on financial stability in those models. In this paper, the financial network serves both as insurance to participants in financial markets, and as a shock transmission mechanism. Financial stability is achieved by maximizing investors long term (period 2) value. Whether insolvency netting does this is questionable, as it brings about two opposing effects.

In order to decide which of the two effects is stronger, one must compare the premium that a patient agent would be willing to pay for such insurance – and then compare it with the expected destruction of value due to domino-effect contagion. Recovery rates in financial sector bankruptcies could be as low as 50% (Denison, Sarkar, and Fleming, 2019), suggesting rather high bankruptcy costs. Such bankruptcy costs are likely to trump any insurance benefits that an interconnected financial network provides.

It is unclear, however, to what extent low recovery rates should be attributed to bankruptcy costs rather than low fundamentals. A bankrupt company may have made bad investment decisions, and as a result the value of its assets is marked down when this information reaches the market. Consider the case of Lehman Brother for example. On the one hand, general creditors recovered only about 31% of their claims,[26] a low number even compared to historical experience. On the other hand, only about 2.5% (roughly $9 billion) are estimated to be the expenses due to the liquidation process (Denison et al., 2019).

It took more than a decade to liquidate Lehman's estate. It seems unlikely that these losses are the result of a prolonged fire-sale. They could in part be due to the loss of Lehman's franchise value, and/or difficulty of asset valuation in a market with severe asymmetric information. But even the latter is not a pure bankruptcy loss, insofar as the loans underlying many of Lehman's structured finance suffered only modest losses. What matters is the global destruction of value, not simple transfers.

The questions raised in this paper are relevant for policy makers around the world. Most countries set up priority rules for the financial system based on incomplete view of the consequence of this legislation. This paper challenges the view that prompted recent changes in U.S. legislation – that distributional effects of netting have no impact on systemic stability. American law is relevant to other advanced economies, as SIFIs choose the jurisdiction which governs their master netting agreements.

From a legal perspective, netting is in conflict with a central principle in bankruptcy law – mandatory stay. By disallowing creditors to collect their assets, mandatory stay aims to allow an orderly liquidation of the estate in order to dispense to creditors maximal value on pro-rata basis. Netting is a form of exclusion from mandatory stay: creditors who are allowed to net are in effect able collect their assets by not paying their liabilities.[27]

[26] 21% if this figure is discounted by corporate bond yield.

[27] There are two main categories of claims on the bankrupt that are excluded from the

Rather than being based on a legal principle, the rationale for the right to net is based on two consequentialist arguments. First, in absence of the right to set-off, OTC dealer markets may suffer a reduction in intermediation services.[28] Second, if a crisis occurs – the inability to set-off will contribute to domino-effect contagion. My paper adds another consequentialist argument to the latter, demonstrating that netting of interbank liabilities may in fact decrease financial stability due to the bank-run channel.

A. Proof of Proposition 1

Theorem 1 in Goldstein and Pauzner (2005) shows that a unique threshold-equilibrium exists, and is the only equilibrium possible. Their result follows from two separate properties of the differential utility function v: (1) single crossing; (2) one-sided strategic complementarities. The latter means that v is decreasing in n (proportion of agents who withdraw early) whenever v is positive. With single crossing, given that all agents use a threshold-equilibrium, then there exists a unique threshold-equilibrium. With one-sided strategic complementarities, any equilibrium must be a threshold-equilibrium.

A. Definitions and Preliminary Results

A mixed strategy for a patient agent i is a function $s_i : [-\epsilon, 1+\epsilon] \rightarrow [0, 1]$, for each possible signal, the agent withdraws early with probability s_i. Let $\tilde{n}(\theta)$ be a random variable with support $[0, 1]$ as the total number of agents that withdraw early in state θ. Let $\tilde{n}_k = \tilde{n}(\theta_)$ be the number of agents who withdraw early in island k. $\tilde{n}_k$ is defined by its CDF $F_{\theta,k}(n)$:

$$F_\theta(n) = \mathbb{P}[\tilde{n}(\theta) \leq n] = \mathbb{P}\left[\lambda + (1-\lambda)\int_{i=0}^{1} s_i(\theta + \epsilon_i)di \leq n\right]$$

estate: claims with right to set-off and secured claims. The main difference between the two categories is that behind the exclusion of a secured claim from mandatory stay stands a firm legal principle, namely security defeats stay – *since the debtor essentially granted the creditor ownership of the security.* Netting negates mandatory stay directly, since there is no transfer of ownership. Moreover, since there is no ownership transfer, such contracts escape negative pledges – meaning third party creditors cannot challenge them in courts (Wood, 2007).

[28] Because taking large positive and negative positions in the trade book would be more risky and thus more costly.

Let $n^x(\theta)$ be the inverse CDF

$$n^x(\theta) = \inf\{n : \ F_\theta(n) \geq x\}$$

A patient agent decides to withdraw early with probability 1, $s_i = 1$, if $\Delta(\cdot, \theta_i) < 0$, withdraw late with probability 1, $s_i = 0$, if $\Delta(\cdot, \theta_i) > 0$, and is indifferent between the two (and therefore could use any mix of pure strategies) if $\Delta(\cdot, \theta_i) = 0$. Recall the definition of Δ (eq. 11-12):

$$\Delta(c, \theta_i, s_{-i}) = \frac{1}{2\epsilon} \int_{\theta_i - \epsilon}^{\theta_i + \epsilon} \underbrace{\left(\int_{n=\lambda}^{1} v(\boldsymbol{\theta}, n) dF_{\boldsymbol{\theta}}(n) \right)}_{\mathbb{E}[v(\theta, \tilde{n}(\theta))|\theta]} d\theta$$
$$v(c, \boldsymbol{\theta}, n) = u(c_2) - [qu(c) + (1-q)u(c_2)]$$

A threshold strategy is characterized by a single number, θ^*, that a patient agent – if she observes a signal higher than that number – will withdraw late, and otherwise she will withdraw early. A threshold-equilibrium is one in which all agents' follow threshold strategies. If all agents follow the same strategy, the proportion of agents who withdraw early in each island n is deterministic.

There are three differences between GP and the current framework: limiting liquidation of the asset at $x \leq 1$

regards consumption of the patient agent in period 2, and limiting the liquidation of the asset at x. Period 2 consumption (eq. 18-21) is given by

$$c_2(\boldsymbol{\theta}) = \begin{cases} \frac{A_k}{1-\min[n, x/c]} & \text{if } A_j \geq D \ \forall j \in \{k, -k\} \\ \frac{A_k - B + \psi[B + A_{-k}]}{1-\min[n, x/c]} & \text{if } A_{-k} < D \ \wedge \ A_k - B + \psi[B + A_{-k}] > D \\ (1-\psi)(A_k + B) & \text{if } A_k < D \ \wedge \ A_{-k} - B + \psi[B + A_k] > D \\ \frac{A_k + \psi A_{-k}}{1+\psi} & \text{else} \end{cases}$$
$$A_k = (1 - \min[nc, x])R(\theta_k)(1 - \gamma_1 \times 1_{n > x/c})(1 - \gamma_2 1_{k \text{ insolvent}})$$

where A_k is the bank's external assets, which may suffer losses due to illiquidity (insufficient period 1 value, $n > x/c$) and insolvency (insufficient period 2 value). Lemma 1 (as in GP) states basic facts about the function $\Delta(c, \theta_i)$. Let

$(\tilde{n}+a)(\theta) = \tilde{n}(\theta+a)$.

Lemma 1 (Lemma 1)**.** *(i)* $\Delta\big(\theta_i, \tilde{n}(\cdot)\big)$ *is continuous in the threshold strategy* θ_i*; (ii)* $\Delta\big(\theta_i{+}a, (\tilde{n}{+}a)(\cdot)\big)$ *is continuous and non-decreasing in* a*; (iii)* $\Delta\big(\theta_i{+}a, (\tilde{n}{+}a)(\cdot)\big)$ *is strictly increasing in* a *if two conditions are satisfied:* $\theta_i + a < \mu_\theta + \epsilon$*; and* $\tilde{n}(\theta) < 1/c$ *with positive probability for* $\theta \in [\theta_i + a - \epsilon, \theta_i + a + \epsilon]$.

Proof of Lemma 1. Note that (i) means Δ is continuous in θ_i given a certain distribution $\tilde{n}(\cdot)$ that doesn't change with θ_i; a change in θ_i only changes the limits of the integration. The integrand $\int_{n=\lambda}^{1} v(\theta, n) dF_\theta(n)$ is bounded because v is bounded. Continuity follows from the fact that $\int_{n=\lambda}^{1} v(\theta, n) dF_\theta(n)$ is Riemann integrable over its domain $[\lambda, 1]$. Riemann integrability follows from the fact that $\int_{n=\lambda}^{1} v(\theta, n) dF_\theta(n)$ can be partitioned into monotone functions, e.g. in case $\gamma_2 = 0$:

$$\int_{n=\lambda_L}^{1} v(\theta, n) dF_\theta(n) = \int_{n=\lambda_L}^{x/c} v(\theta, n) dF_\theta(n) + \int_{n=x/c}^{1} v(\theta, n) dF_\theta(n)$$

and each of those is Riemann integrable (because it is monotone, as we are not changing the distribution $\tilde{n}(\cdot)$, and v is monotone on each sub-interval), and thus each is continuous in θ_i. In case $\gamma_2 > 0$, partition is into potentially 9 parts, each of which is monotone in θ. The sum of two or more continuous functions is continuous. (ii) Continuity with respect to a follows because v is bounded and Δ is an integral over a segment $[\theta_i - \epsilon, \theta_i + \epsilon]$. Note that the distribution $\tilde{n}$ is only shifted by a constant a. Thus if we compare any θ with $\theta + a$, $\tilde{n}$ is identical, but at $\theta + a$ fundamentals are higher, so $R(\theta)$ is higher and consequently v. Since v is, ceteris paribus, non-decreasing in θ, so is Δ. (iii) In continuation to (ii), if the two conditions are kept then v is strictly increasing for at least part of the interval $[\theta_i - \epsilon, \theta_i + \epsilon]$, thus its integral is strictly increasing. This completes the proof. $\square$

Definition 5 (Strategic Complementarities)**.** *A game is said to exhibit strategic complementarities if the incentive of agent* i *to increase* s_i *is increasing in* s_{-i}*. More precisely, if* $s_i \geq \hat{s}_i$ *and* $s_{-i} \geq \hat{s}_{-i}$ *– which implies* $F_\theta(n) \leq \hat{F}_\theta(n)$ *– we have*

$$u(s_i, s_{-i}, \theta) - u(\hat{s}_i, s_{-i}, \theta) \geq u(s_i, \hat{s}_{-i}, \theta) - u(\hat{s}_i, \hat{s}_{-i}, \theta)$$

A game is said to exhibit one-sided strategic complementarities if the incentive of agent i to increase s_i is increasing in s_{-i} whenever that incentive is positive.

Lemma 2 (Strategic Complementarities)**.** *The game played by patient agents exhibits strategic complementarities.*

Proof of Lemma 2. Simplifying v yields $v(n, \cdot) = q\big(u(c_2) - u(c)\big)$. Observe that the probability of consumption in period 1 is certain ($q = 1$) for $n \leq x/c$, and is decreasing from 1 to $\frac{x}{c}$ for $n \in (x/c, 1]$. Moreover, due to bankruptcy costs, for a fixed $\boldsymbol{\theta}$, A_k may have at most two discontinuities: one exactly at $n = x/c$; the other potentially at a point $n < x/c$ (since A_k decreases in n and may hit insolvency while still liquid). Thus, when $n > x/c$, A_k is fixed. $\frac{\partial v}{\partial n} < 0$ for $n \in [\lambda, x/c]$, and $\frac{\partial v}{\partial n} > 0 (< 0)$ in n for $n \in (x/c, 1]$ when $u(c_2) - u(c) < 0 \ (> 0)$. In case $v(n, \cdot) > 0$ for $n \in (x/c, 1)$, then v is decreasing because $c_2 > c$ and because q is decreasing.

This proves strategic complementarities in a single island k, and for a fixed n. Observe that $\frac{\partial A_{-k}}{\partial n} \leq 0$, and that $\frac{\partial c_2}{\partial A_{-k}} \geq 0$. Thus due to the chain rule: $\frac{\partial c_2}{\partial n} \leq 0$, increasing the incentive to run on the bank. $\square$

Lemma 3 (Single Crossing)**.** *Keeping $\boldsymbol{\theta}$ fixed, there exists $n' \in (\lambda, 1)$ such that $v(n, \boldsymbol{\theta}) \geq 0 \ \forall n < n'$ and $v(n, \boldsymbol{\theta}) \leq 0 \ \forall n \geq n'$.*

Proof of Lemma 3. Single crossing is implied by strategic complementarities. If $v(n, \cdot) < 0$ for some $n \in (x/c, 1)$, it is negative throughout, so that v may cross (up to discontinuity) from positive to negative at most once. $\square$

The statement in Lemma 3 is for a fixed a $\boldsymbol{\theta}$. Observe that if all agents follow a threshold strategy, $\frac{\partial n}{\partial \theta_k} \leq 0$, and therefore $\frac{\partial c_2}{\partial \theta_j} \geq 0 \ \forall j \in \{k, -k\}$: better fundamentals in either island imply (weakly) higher consumption in period 2. Thus varying θ over the interval $[\theta_i - \epsilon, \theta_i + \epsilon]$, v could cross zero (up to discontinuity) at most once.

B. *Unique Threshold-equilibrium*

Proof of Proposition 1. The proof follows GP Proposition 1 closely. The only difference is that in my model v crosses zero only once up to a discontinuity, which doesn't alter the details of the proof much.

A threshold-equilibrium with threshold θ^* exists iff, given that all other agents use therehold strategies θ^*, each agent finds it optimal to withdraw early when she observes a signal below θ^*, and late if above it:

$$\Delta\big(c, \theta_i, \tilde{n}(\cdot, \theta^*)\big) < 0 \ \forall \theta_i < \theta^* \tag{22}$$

$$\Delta\big(c, \theta_i, \tilde{n}(\cdot, \theta^*)\big) > 0 \ \forall \theta_i > \theta^* \tag{23}$$

By continuity (Lemma 1i), a patient agent is indifferent when she observes θ^*

$$\Delta\big(c, \theta^*, \tilde{n}(\cdot, \theta^*)\big) = 0 \tag{24}$$

I now show given that eq. 24 holds, eqs. 22-23 hold as well. Assume $\theta_i < \theta^*$, and partition each interval $[\theta^* - \epsilon, \theta^* + \epsilon]$ and $[\theta_i - \epsilon, \theta_i + \epsilon]$ to two complementary intervals: the intersection between the two, and its complement for each interval.

Let $Y = [\theta_i - \epsilon, \theta_i + \epsilon] \cap [\theta^* - \epsilon, \theta^* + \epsilon]$ be the intersection (possibly empty) of the interval over which $\Delta\big(c, \theta_i, \tilde{n}(\cdot, \theta^*)\big)$ and $\Delta\big(c, \theta^*, \tilde{n}(\cdot, \theta^*)\big)$ are evaluated, and two disjoint intervals $Z^i = [\theta_i - \epsilon, \theta_i + \epsilon] \setminus Y$ and $Z^* = [\theta^* - \epsilon, \theta_* + \epsilon] \setminus Y$. This yields

$$\Delta\big(c, \theta^*, \tilde{n}(\theta)\big) = \frac{1}{2\epsilon} \int_{\theta \in Y} v\big(\theta, n(\theta)\big) d\theta + \frac{1}{2\epsilon} \int_{\theta \in Z^*} v\big(\theta, n(\theta)\big) d\theta \tag{25}$$

$$\Delta\big(c, \theta_i, \tilde{n}(\theta)\big) = \frac{1}{2\epsilon} \int_{\theta \in Y} v\big(\theta, n(\theta)\big) d\theta + \frac{1}{2\epsilon} \int_{\theta \in Z^i} v\big(\theta, n(\theta)\big) d\theta \tag{26}$$

Equation 25 must equal zero by assumption. Because $\theta_i < \theta^*$, the part integrated over the complement interval is non-positive (due to single crossing, Lemma 3). When comparing the second part in each equation (the complement of intersection), we must have

$$\frac{1}{2\epsilon} \int_{\theta \in Z^*} v\big(\theta, n(\theta)\big) d\theta > \frac{1}{2\epsilon} \int_{\theta \in Z^i} v\big(\theta, n(\theta)\big) d\theta$$

This implies eq. 22 must be negative. A diametrical argument holds for eq. 23. Due to strategic complementarities (Lemma 2), GP's argument follows through in full.

$\square$

B. Proof of Proposition 4

Proof. We are interested in the effect of netting on the probability of runs. The proof utilizes the implicit function theorem on this indifference condition to show the effect of the interbank mutual claim B on the probability of runs θ^*. The result states that if $\gamma_2 = 0$ then $\frac{\partial \theta^*}{\partial B} < 0$. Since netting decreases B, the result shows netting increases the probability of bank runs.

Equation 11 is the indifference condition that determines the optimal threshold signal θ^*. In equilibrium, all agents use the threshold strategy θ^*, and their indifference condition is equal to zero.

$$\Delta(\theta^*, \cdot) = \frac{1}{2\epsilon} \int_{\theta^*-\epsilon}^{\theta^*+\epsilon} \underbrace{\left(\int_{n=\lambda}^{1} v(\boldsymbol{\theta}, n) dF_{\boldsymbol{\theta}}(n) \right)}_{\mathbb{E}[v(\theta,n)|\theta_i=\theta^*]} = 0$$

Recall that $v = q\big(u(c_2) - u(c)\big)$. Thus we can write

$$\Delta(\theta^*, \cdot) = \mathbb{E}[q\big(u(c_2) - u(c)\big)|\theta_i = \theta^*] \tag{27}$$

It is clear from eq. 27 that $\frac{\partial \Delta(\theta^*, \cdot)}{\partial \mathbb{E}[u(c_2)|\theta_i=\theta^*]} > 0$. Increasing expected utility from consumption in period 2 increases the incentive to withdraw late. Note also that $\frac{\partial \Delta(\theta^*, \cdot)}{\partial \theta^*} \geq 0$ because changing θ^* varies the limits of the integration leaving the distribution of early withdrawals on the interval $[\theta^* - \varepsilon, \theta^* + \varepsilon]$ unchanged. Due to the implicit function theorem, we have

$$\frac{\partial \theta^*}{\partial \mathbb{E}[u(c_2)]} < 0 \tag{28}$$

Now consider the effect of B on $\mathbb{E}[u(c_2)|\theta_i = \theta^*]$, given that $\gamma_2 = 0$. In equilibrium, every state $\boldsymbol{\theta}$ is associated with c_2 in island k and $-k$. Denote by $t = t(\boldsymbol{\theta}) \in \mathbb{R}$ the transfer from k to $-k$ that is due to the interbank connection B; it is either:

- $t > 0$ $-k$ insolvent; if k is also insolvent, it has a higher return than $-k$;
- $t < 0$ k insolvent; if $-k$ is also insolvent, it has a higher return than k

Observe that, due to symmetry of $f_{\boldsymbol{\theta}}$, for every state $\boldsymbol{\theta}^+ = (\theta, \theta')$ there exists $\boldsymbol{\theta}^- = (\theta', \theta)$ such that: (1) $f(\boldsymbol{\theta}^+) = f(\boldsymbol{\theta}^-)$; (2) $t(\boldsymbol{\theta}^+) = -t(\boldsymbol{\theta}^-)$. Generally, whenever $t > 0$ (< 0) period 2 consumption in island k ($-k$) is higher than that of island $-k$ (k).

Let Θ^+ be a subspace of Θ such that $(\theta, \theta') \in \Theta^+$ iff $t(\theta', \theta) > 0$; Θ^+ is a set of unique states of the world (up to island names). Denote by Θ^- its mirror image sub-space $\Theta^- = \{(\theta', \theta) : (\theta, \theta') \in \Theta^+\}$, and write c_2 as the consumption in island k in case $B = 0$. When computing expected utility differential in island k we have

$$
\begin{aligned}
\Delta(\theta^*, \cdot) = \quad & \int_{\boldsymbol{\theta}^+ \in \Theta^+} q\big(u(c_2 + t) - u(c)\big) d\boldsymbol{\theta}^+ f(\boldsymbol{\theta}^+ | \theta_i = \theta^*) \\
+ \quad & \int_{\boldsymbol{\theta}^- \in \Theta^-} q\big(u(c_2 - t) - u(c)\big) f(\boldsymbol{\theta}^- | \theta_i = \theta^*) d\boldsymbol{\theta}^- \\
+ \quad & \int_{\boldsymbol{\theta} \in \Theta \setminus (\Theta^+ \cup \Theta^-)} q\big(u(c_2) - u(c)\big) f(\boldsymbol{\theta} | \theta_i = \theta^*) d\boldsymbol{\theta} \\
= \quad & \int_{\boldsymbol{\theta}^+ \in \Theta^+} \Big(q(\boldsymbol{\theta}^+)\big(u(c_2(\boldsymbol{\theta}^+) + t) - u(c)\big) \\
+ \quad & q(\boldsymbol{\theta}^-)\big(u(c_2(\boldsymbol{\theta}^-) - t) - u(c)\big)\Big) f(\boldsymbol{\theta}^+ | \theta_i = \theta^*) d\boldsymbol{\theta}^+ \\
+ \quad & \int_{\boldsymbol{\theta} \in \Theta \setminus (\Theta^+ \cup \Theta^-)} q\big(u(c_2) - u(c)\big) f(\boldsymbol{\theta} | \theta_i = \theta^*) d\boldsymbol{\theta}
\end{aligned}
\tag{29}
$$

Note that when $t > 0$, the fundamentals in island k are lower than in $-k$. Since agents observe a signal informing them on the common factor of θ_k, $n_k = n_{-k}$ in all states, therefore the probability of consumption in period 1 is always the same in both islands. Also, observe that varying B can only change q via θ^*, as it doesn't affect liquid assets x. Thus, keeping θ^* fixed, q is unchanged when varying B. We need to show

$$
q(\boldsymbol{\theta}^+) u(c_2(\boldsymbol{\theta}^+) + t) + q(\boldsymbol{\theta}^-) u(c_2(\boldsymbol{\theta}^-) - t) \geq q(\boldsymbol{\theta}^+) u(c_2(\boldsymbol{\theta}^+)) + q(\boldsymbol{\theta}^-) u(c_2(\boldsymbol{\theta}^-))
$$

which is true because: (1) $q(\boldsymbol{\theta}^+) = q(\boldsymbol{\theta}^-)$; (2) $c_2(\boldsymbol{\theta}^+) + t \leq c_2(\boldsymbol{\theta}^-) - t$. Moreover, in the limit as $B \to \infty$, both banks are in effect consolidated, so we have

$$
\lim_{B \to \infty} t = \frac{c_2(\boldsymbol{\theta}^-) - c_2(\boldsymbol{\theta}^+)}{2}
$$

Because agents are risk averse,

$$u(c_2(\boldsymbol{\theta}^+) + t) + u(c_2(\boldsymbol{\theta}^-) - t) \geq u(c_2(\boldsymbol{\theta}^+)) + u(c_2(\boldsymbol{\theta}^-))$$

Finally, $\frac{\partial \mathbb{E} u(c_2)}{\partial B} > 0$, so together with eq. 28 we get the desired result.

$\square$

A. Conditions for using the Implicit Function Theorem

The main condition for using the implicit function theorem is continuous non-zero first derivatives of Δ in a small open ball around the values in the parameter space that make $\Delta(\theta^*, \cdot) = 0$. This condition holds for $\frac{\partial \Delta}{\partial \theta^*}$ due to Lemma 1. Varying B doesn't change the limits of the integration of the integral in Δ. The partial derivative $\frac{\partial \Delta}{\partial B}$ is continuous because $\frac{\partial v}{\partial B}$ is continuous. The latter is the case because varying B doesn't affect outside assets A_k, A_{-k}; rather it varies only the interbank transfer t.

C. Proof of Proposition 2

First pass: in the limit as $\epsilon \to 0$, fixed θ^*.

Let P be the unconditional probability that $\theta < \theta^*$: $P = \int_{-\infty}^{\theta^*} f_\theta d\theta$. Then the unconditional expectation of utility in period 0 is given by:

$$\begin{aligned} &= P\lambda u(c) + (1-\lambda)\mathbb{E}[u(c_2)|\theta > \theta^*] + (1-P)\tfrac{x}{c}u(c) + \left(1 - \tfrac{x}{c}\right)(1-\lambda)\mathbb{E}[u(c_2)|\theta < \theta^*] \\ &= P\lambda u(c) + (1-\lambda)\mathbb{E}[u(c_2)] + \tfrac{x}{c}\mathbb{E}[u(c) - (1-\lambda)u(c_2)|\theta < \theta^*] \end{aligned}$$

All terms here are positive. The last term is positive because, in equilibrium, expected utility from withdrawing late is lower than expected utility from withdraw early when $\theta < \theta^*$. The first derivative of this object is:

$$= P\lambda u'(c) + (1-\lambda)\mathbb{E}[u'(c_2)]\left(-\frac{x(1-x)c}{c-x}\right) - \frac{x}{c^2}\mathbb{E}[u(c) - (1-\lambda)u(c_2)|\theta < \theta^*] + \frac{x}{c}\mathbb{E}[u'(c) + (1-\lambda)\frac{x(1-x)c}{c-x}$$

We have two positive terms and two negative terms. Each of the positive terms is decreasing, and each of the negative

Brute force did not yield the required outcome. Looking at the problem from a different perspective.

The deposit rate which maximizes expected utility, conditional on no run:

$$\max_c \int_{\theta^*}^{\infty} \left[\lambda u(c) + (1-\lambda) u\left(\frac{1-\lambda c}{1-\lambda} R(\theta)\right)\right] d\theta \tag{30}$$

The first order condition is given by:

$$\int_{\theta^*}^{\infty} \lambda \left[u'(c) - u'\left(\frac{1-\lambda c}{1-\lambda} R(\theta)\right) R(\theta)\right] d\theta - \frac{\partial \theta^*}{\partial c}\left[\lambda u(c) + (1-\lambda) u\left(\frac{1-\lambda c}{1-\lambda} R(\theta^*)\right)\right] = 0 \tag{31}$$

The second order condition is given by:

$$\begin{aligned}
&\int_{\theta^*}^{\infty} \lambda \left[u''(c) + \frac{\lambda}{1-\lambda} u''\left(\frac{1-\lambda c}{1-\lambda} R(\theta)\right) R(\theta)^2\right] d\theta \\
&\quad -2\frac{\partial \theta^*}{\partial c}\lambda \left[u'(c) - u'\left(\frac{1-\lambda c}{1-\lambda} R(\theta^*)\right) R(\theta^*)\right] \\
&\quad -\frac{\partial^2 \theta^*}{\partial c^2}\left[\lambda u(c) + (1-\lambda) u\left(\frac{1-\lambda c}{1-\lambda} R(\theta^*)\right)\right]
\end{aligned}$$

In principle this second order condition can be positive, because $\frac{\partial^2 \theta^*}{\partial c^2}$ can be shown to be negative. However, this would require the second order term to overturn two strongly negative terms that are first order.

The optimal value of the deposit rate, conditional on a bank-run, is roughly $c \approx (1-x)\mathbb{E}R/(1-x/c)$, which makes consumption of agents who managed to get cash out the same as those who didn't (the exact value must take account that a higher deposit rate means more impatient consumers losing all value). This can be shown to be lower than the deposit rate which is optimal conditional on no run.

Solving this is a bit like the monopoly problem I'm solving for Andre. It can be calculated from the welfare loss

Given that we have a unique value of c that maximizes ex-ante utility conditional on no runs

D. Technical Appendix

This appendix describes in detail my simulation methodology. I make assumptions about the utility, technology and the distribution of θ and ν_k to simplify expressions of the function Δ (expected differential utility):

1. The utility function exhibits constant relative risk aversion:

$$u(c_t) = 1 - \exp(-\alpha\ c_t)$$

2. Returns on total assets are assumed to be linear in θ:

$$R_k = R\ (\theta_k, x) = xR_f + (1-x)\theta_k$$

   where R_f is the gross risk free rate and θ the return on the long asset.

3. The common and idiosyncratic components of returns are all pairwise independent and distributed normally

$$\begin{pmatrix} \theta \\ \nu_k \\ \nu_{-k} \end{pmatrix} = N\left(\begin{bmatrix} \mu_\theta \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\theta^2 & 0 & 0 \\ 0 & \sigma_\nu^2 & 0 \\ 0 & 0 & \sigma_\nu^2 \end{bmatrix} \right)$$

Recall $\theta_k = \theta + \nu_k$. Let ϕ and Φ be (respectively) the pdf and CDF of a normal random variable. Recall that (in island k) the return θ_k and the level of period 2 assets A_k are given by:

$$A_k(\theta_k) = (1 - \min[nc, x])R(\theta_k)\Gamma$$
$$R_k = R(\theta_k, x) = xR_f + (1-x)\theta_k$$

It is useful to work with their inverse:

$$A^{-1} = \theta_k(A, x, c, \theta) = \begin{cases} \left(\frac{A}{1-\tilde{\gamma}_1} \frac{1}{1-\min[nc,x]} - xR_f \right) \frac{1}{1-x} & \text{if } A \geq D \\ \left(\frac{A}{\Gamma} \frac{1}{1-\min[nc,x]} - xR_f \right) \frac{1}{1-x} & \text{if } A < D \end{cases}$$

$$R^{-1} = \theta(R, x) = (R - xR_f)/(1-x)$$

For a given θ, the state-space is two dimensional: one axis for ν_k and another for ν_{-k}. I partition the state-space into 4 regions, based on whether k and $-k$ are bankrupt in period 2. Region 4 is sub-partioned to 3, depending on whether k $(-k)$ is insolvent due to the interbank connection (see Table I).

The boundaries between regions are presented in table IV. These are used to partition the integral $\int_{\theta_k \in \mathbb{R}} \int_{\theta_{-k} \in \mathbb{R}} \cdots$ into sums of integrals on mutually exclusive sub-spaces – one sum for each region:

$$\Delta\ (\theta_i = \theta^*, \theta^*) = \sum_{m \in \{1,2,3,4a,4b,4c\}} \Delta_m \tag{32}$$

$$\Delta(\theta^*, \theta_i, \cdot) = \int_{\theta=\theta_i-\epsilon}^{\theta_i+\epsilon} \int_{\theta_k \in \mathbb{R}} \int_{\theta_{-k} \in \mathbb{R}} q\big(u(c_2) - u(c)\big) f_{\boldsymbol{\theta}}(\theta_k, \theta_{-k}|\theta) d\theta_k d\theta_{-k}$$

Table IV. State-space partition: boundaries

Region	θ_k	θ_{-k}
Θ_1	$\theta_j \geq \overbrace{\left(\frac{D}{1-\tilde{\gamma}_1} \frac{1}{1-\min[nc, x]} - xR_f\right) \frac{1}{1-x}}^{\hat{\theta}_1}$ for $j \in \{k, -k\}$	
Θ_2	$\theta_k \geq \overbrace{\left(\frac{D + B(1-\psi) - \psi A_{-k}}{1-\tilde{\gamma}_1} \frac{1}{1-\min[nc, x]} - xR_f\right) \frac{1}{1-x}}^{\hat{\theta}_2(A_{-k})}$	$\theta_{-k} < \hat{\theta}_1$
Θ_3	$\theta_k < \hat{\theta}_1$	$\theta_{-k} \geq \hat{\theta}_2(A_k)$
Θ_{4a}	$\theta_j < \hat{\theta}_1$ for $j \in \{k, -k\}$	
Θ_{4b}	$\theta_k < \hat{\theta}_2(A_{-k})$	$\theta_{-k} < \hat{\theta}_1$
Θ_{4c}	$\theta_k < \hat{\theta}_1$	$\theta_{-k} < \hat{\theta}_2(A_k)$

A. Region 1

Both banks are solvent, $A_k \geq D \wedge A_{-k} \geq D$. Bank $-k$ is solvent, given θ, with probability $\Big(1 - \Phi\big(\hat{\theta}_1,\ \theta,\ \sigma_\nu\big)\Big)$. Otherwise bank $-k$ has no effect on (period 2) expected utility in island k (given θ), which is given by

$$q(\theta)\int_{\theta_k=\hat{\theta}_1}^{\infty} u\left(\frac{A_k}{1-\min[n,x/c]}\right)\phi(\theta_k,\theta,\sigma_\nu)d\theta_k$$

Due to CARA utility we have

$$= -\frac{1}{\sigma_\nu\sqrt{2\pi}}\int_{\theta_k=\hat{\theta}_1}^{\infty} e^{-\alpha\frac{A_k}{1-\min[n,x/c]}}e^{-\frac{1}{2\sigma_\nu^2}(\theta_k-\theta)}d\theta_k$$
$$= -\frac{1}{\sigma_\nu\sqrt{2\pi}}\int_{\theta_k=\hat{\theta}_1}^{\infty} e^{-\alpha\left(c-\frac{c-1}{1-\min[n,x/c]}\right)\left(xR_f+(1-x)\theta_k\right)\left(1-\tilde{\gamma}_1\right)}e^{-\frac{1}{2\sigma_\nu^2}(\theta_k-\theta)}d\theta_k$$

The assumption of CARA utility with normal distribution, together with the linear (in θ_k) functional form of returns R_k, means we can complete the square to get a shifted mean normal distribution (because u is exponential function linear in θ_k). Let b_1 be the coefficient on θ_k due to the utility function, and a_1 the free coefficient:

$$a_1 = -\alpha\left(c - \frac{c-1}{1-\min[n,x/c]}\right)xR_f(1-\tilde{\gamma}_1)$$
$$b_1 = -\alpha\left(c - \frac{c-1}{1-\min[n,x/c]}\right)(1-x)(1-\tilde{\gamma}_1)$$

This results in

$$\begin{aligned} &= -e^{a_1+b_1\theta+\sigma_\nu^2(b_1)^2/2}\frac{1}{\sigma_\nu\sqrt{2\pi}}\int_{\theta_k\in[\hat{\theta}_1,\infty)} e^{-\frac{1}{2\sigma_\nu^2}(\theta_k-(\theta+b_1\sigma_\nu^2))}d\theta_k \\ &= -e^{a_1+b_1\theta+\sigma_\nu^2(b_1)^2/2}\Big(1-\Phi\big(\hat{\theta}_1,\ \theta+b_1\sigma_\nu^2,\ \sigma_\nu\big)\Big)\end{aligned}$$

Thus we have

$$Z_1(\theta) = \Big[1-\Phi(\hat{\theta}_1,\ \theta,\sigma_\nu) - e^{a_1+b_1\theta+(b_1\sigma_\nu)^2/2}\Big(1-\Phi\big(\hat{\theta}_1,\ \theta+b_1\sigma_\nu^2,\ \sigma_\nu\big)\Big]\Big)\Big(1-\Phi\big(\hat{\theta}_1,\ \theta,\ \sigma_\nu\big)\Big)$$

$$= \Big\{1-\Phi(\hat{\theta}_1,\ \theta,\sigma_\nu) - \exp\Big(-\alpha\left(c-\frac{c-1}{1-\min[n,x/c]}\right)\big(xR_f+(1-x)\theta\big)\big(1-\tilde{\gamma}_1\big)$$

$$+\left[\alpha\left(c-\frac{c-1}{1-\min[n,x/c]}\right)(1-x)\big(1-\tilde{\gamma}_1\big)\sigma_\nu\right]^2/2\Big)\times$$

$$\Big(1-\Phi\big(\hat{\theta}_1,\ \theta-\alpha(1-x)\left(c-\frac{c-1}{1-\min[n,x/c]}\right)(1-\tilde{\gamma}_1)\sigma_\nu.^2,\ \sigma_\nu\big)\Big)$$

$$\Big\}\times\Big(1-\Phi\big(\hat{\theta}_1,\ \theta,\ \sigma_\nu\Big)$$

$$\Delta_1(\theta^*)=\int_{\theta=\theta^*-\epsilon}^{\theta^*+\epsilon} q(\theta)Z_1(\theta)d\theta$$

B. Region 2

Bank k solvent, $-k$ insolvent, $A_k+\psi(A_{-k}+B)\geq D+B \wedge A_{-k}<D$. The lower boundary of the integral on θ_{-k}, $A^{-1}(\frac{1}{\psi}(B(1-\psi)+D-A_k))$ is derived from the condition that k stays solvent, i.e. the inequality $A_k+\psi(A_{-k}+B)\geq D+B$. Period 2 consumption in island k depends on how deep is $-k$'s insolvency, and is given by

$$c_2(\boldsymbol{\theta}\ |\boldsymbol{\theta}\in\Theta_2)=\left(c-\frac{c-1}{1-\min[n,x/c]}\right)\big(xR_f(1-\tilde{\gamma}_1+\psi\Gamma)+$$
$$(1-x)((1-\tilde{\gamma}_1)\theta_k+\psi\Gamma\theta_{-k})\big)-\frac{(1-\psi)B}{1-\min[n,x/c]}$$

We can disentangle the component due to the interbank connection in $u(c_2(\boldsymbol{\theta}\ |\boldsymbol{\theta}\in\Theta_2))$, with the coefficients (for completing the square) being:

$$a_2^k=-\alpha\left(\left(c-\frac{c-1}{1-\min[n,x/c]}\right)xR_f(1-\tilde{\gamma}_1)-\frac{(1-\psi)B}{1-\min[n,x/c]}\right)$$
$$a_2^{-k}=-\alpha\ \psi\left(c-\frac{c-1}{1-\min[n,x/c]}\right)xR_f\Gamma$$
$$b_2^k=-\alpha\ \left(c-\frac{c-1}{1-\min[n,x/c]}\right)(1-x)(1-\tilde{\gamma}_1)$$
$$b_2^{-k}=-\alpha\ \psi\left(c-\frac{c-1}{1-\min[n,x/c]}\right)(1-x)\Gamma$$

When completing the square on the non-constant component (belonging to k), this yields

$$Z_2(\theta,A_{-k})=e^{a_2^k+b_2^k\theta+(b_2^k\sigma_\nu)^2/2}$$

$$\frac{1}{\sigma_\nu\sqrt{2\pi}}\int_{\theta_k=\hat{\theta}_2(A_{-k})}^{\infty} e^{-\frac{1}{2\sigma_\nu^2}(\theta_k-(\theta+\sigma_\nu^2 b_2^k))} d\theta_k$$

$$= e^{a_2^k+b_2^k\theta+(b_2^k\sigma_\nu)^2/2}$$

$$\Big(1-\Phi\big(\hat{\theta}_2(A_{-k}),\ \theta+\sigma_\nu^2 b_2^k,\ \sigma_\nu\big)\Big)$$

(period 2) expected utility (given θ) in island k is given by

$$\begin{aligned}\Delta_2(\theta^*) &= \int_{\theta=\theta^*-\epsilon}^{\theta^*+\epsilon} q(\theta)\int_{\theta_{-k}=-\infty}^{\hat{\theta}_1}\Big[1-\Phi(\hat{\theta}_2(A_{-k}),\theta,\sigma_\nu)-Z_2(\theta,A_{-k})\times \\ &\qquad \exp(a_2^{-k}+b_2^{-k}\theta_{-k})\Big]\phi(\theta_k,\theta,\sigma_\nu)d\theta_k d\theta \\ &= \int_{\theta=\theta^*-\epsilon}^{\theta^*+\epsilon} q(\theta)\Big\{\int_{\theta_{-k}=-\frac{x}{1-x}}^{\hat{\theta}_1}\Big[1-\Phi(\hat{\theta}_2(A_{-k}),\theta,\sigma_\nu)-Z_2(\theta,A_{-k})\times \\ &\qquad \exp(a_2^{-k}+b_2^{-k}\theta_{-k})\Big]\phi(\theta_k,\theta,\sigma_\nu)d\theta_k + \\ &\qquad \Big[1-\Phi(\hat{\theta}_2(0),\theta,\sigma_\nu)-Z_2(\theta,0)\Big]\ \Phi(-\frac{x}{1-x},\theta,\sigma_\nu)\Big\}d\theta\end{aligned}$$

In this region we cannot use completion to square because of the expression Z_2 inside the integral on θ_k. Moreover, we cannot use integral2 because the boundary of the integral on θ_k depends on θ, thus we have to use nested integrals.

C. Region 3

Bank k insolvent, $-k$ solvent, $A_k < D\ \wedge\ A_{-k}+\psi(A_k+B)\geq D+B$. The lower boundary of the integral on θ_{-k}, $A^{-1}(B(1-\psi)+D-\psi A_k))$ is derived from the condition that $-k$ stays solvent, i.e. the inequality $A_{-k}+\psi(A_k+B)\geq D+B$. Period 2 consumption in island k doesn't depend on $-k$'s assets, but the boundary of the integral does depend on it, as well as the probability that $-k$ remains solvent. Those are given by

$$\begin{aligned}c_2(\boldsymbol{\theta}\ |\boldsymbol{\theta}\in\Theta_3) = &\quad (1-\psi)\Big(\Big(c-\tfrac{c-1}{1-\min[n,x/c]}\Big)\left(xR_f+(1-x)\theta_k\right)\Gamma+\tfrac{B}{1-\min[n,x/c]}\Big) \\ Z_3(A_k) = &\quad 1-\Phi\Big(\hat{\theta}_2(A_k),\ \theta,\ \sigma_\nu\Big)\end{aligned}$$

The coefficients in this region are

$$
\begin{aligned}
a_3^k &= -\alpha(1-\psi)\left(c - \frac{c-1}{1-\min[n, x/c]}\right) x R_f \Gamma \\
a_3^{-k} &= -\alpha(1-\psi)\frac{B}{1-\min[n, x/c]} \\
b_3^k &= -\alpha(1-\psi)\left(c - \frac{c-1}{1-\min[n, x/c]}\right)(1-x)\Gamma
\end{aligned}
$$

(period 2) expected utility (given θ) in island k is given by

$$
\begin{aligned}
\Delta_3(\theta^*) = & \int_{\theta=\theta^*-\epsilon}^{\theta^*+\epsilon} q(\theta) \int_{\theta_k=-\infty}^{\hat{\theta}_1} Z_3(A_k) \times u\left(c_2(\boldsymbol{\theta} \mid \boldsymbol{\theta} \in \Theta_2)\right) \phi(\theta_k, \theta, \sigma_\nu) d\theta_k d\theta \\
= & \int_{\theta=\theta^*-\epsilon}^{\theta^*+\epsilon} q(\theta) \Big\{ \int_{\theta_k=-\frac{x}{1-x}}^{\hat{\theta}_1} Z_3(A_k) \times u\left(c_2(\boldsymbol{\theta} \mid \boldsymbol{\theta} \in \Theta_2)\right) \phi(\theta_k, \theta, \sigma_\nu) d\theta_k \\
+ & \quad Z_3(0)\big(1-\exp(a_3^{-k})\big)\Phi\left(-\tfrac{x}{1-x}, \theta, \sigma_\nu\right)\Big\} d\theta
\end{aligned}
$$

In this region we cannot use completion to square because of the expression Z_3 inside the integral on θ_k. Moreover, we cannot use reduce further the integral because the boundary depends on θ_k, thus we have to use nested integrals.

C.1. Region 4a

Both banks are insolvent, and both would have been insolvent even when the other bank is solvent: $A_k < D \ \wedge \ A_{-k} < D$. Period 2 consumption in island k is given by:

$$
c_2(\boldsymbol{\theta} \mid \boldsymbol{\theta} \in \Theta_{4a}) = \frac{A_k + \psi A_{-k}}{(1+\psi)(1-\min[n, x/c])}
$$

One can decompose expected utility from withdrawing late in this region to two components independent from one another,

$$
\begin{aligned}
&\int_{\theta=\theta_i-\epsilon}^{\theta_i+\epsilon} q(\theta) \int_{\theta_k=-\infty}^{\hat{\theta}_1} \int_{\theta_{-k}=-\infty}^{\hat{\theta}_1} u(c_2(\boldsymbol{\theta} \mid \boldsymbol{\theta} \in \Theta_{4a}))\phi(\theta_k, \theta, \sigma_\nu)\phi(\theta_{-k}, \theta, \sigma_\nu) d\theta_{-k} d\theta_k d\theta = \\
&\qquad \int_{\theta=\theta_i-\epsilon}^{\theta_i+\epsilon} q(\theta)\Big(\Phi(\hat{\theta}_1, \theta, \sigma_\nu)^2 - \\
&\int_{\theta_k=-\frac{x}{1-x}}^{\hat{\theta}_1} e^{-\frac{\alpha A_k}{(1+\psi)(1-\min[n,x/c])}} \phi(\theta_k, \theta, \sigma_\nu) d\theta_k \int_{\theta_{-k}=-\frac{x}{1-x}}^{\hat{\theta}_1} e^{-\frac{\alpha \psi A_{-k}}{(1+\psi)(1-\min[n,x/c])}} \phi(\theta_{-k}, \theta, \sigma_\nu) d\theta_{-k}\Big) d\theta
\end{aligned}
$$

and then complete the square for each. The expression $\Phi(\hat{\theta}_1, \theta, \sigma_\nu)^2$ comes from adding 1 to the utility function. The lower boundary of the integral is $-\infty$ for the constant component of utility, but is $1\frac{x}{1-x}$ for the one depending on assets, since in that case assets are simply zero. The coefficients for completing the square are:

$$a_4^k = -\alpha\left(c - \frac{c-1}{1-\min[n, x/c]}\right)x\Gamma \times \frac{1}{1+\psi}$$
$$a_4^{-k} = a_4^k \times \psi$$
$$b_4^k = -\alpha\left(c - \frac{c-1}{1-\min[n, x/c]}\right)(1-x)\Gamma\frac{1}{1+\psi}$$
$$b_4^{-k} = b_4^k \times \psi$$

This yields

$$\Delta_{4a} = \int_{\theta=\theta_i-\epsilon}^{\theta_i+\epsilon} q(\theta)\Big\{\Phi(\hat{\theta}_1, \theta, \sigma_\nu)^2 -$$
$$\left[\left(\Phi(\hat{\theta}_1,\ \theta + b_4^k\sigma_\nu^2,\ \sigma_\nu) - \Phi(-\frac{x}{1-x},\ \theta + b_4^k\sigma_\nu^2,\ \sigma_\nu)\right) \times e^{a_4^k + b_4^k\theta[(b_4^k)^2\sigma_\nu]^2/2} + \Phi(-\frac{x}{1-x}, \theta, \sigma_\nu)\right] \times$$
$$\left[\left(\Phi(\hat{\theta}_1,\ \theta + b_4^{-k}\sigma_\nu^2,\ \sigma_\nu) - \Phi(-\frac{x}{1-x},\ \theta + b_4^{-k}\sigma_\nu^2,\ \sigma_\nu)\right) \times e^{a_4^{-k} + b_4^{-k}\theta[(b_4^{-k})^2\sigma_\nu]^2/2} + \Phi(-\frac{x}{1-x}, \theta, \sigma_\nu)\right]\Big\}d\theta$$

This expression depends only on θ, reducing the triple to a single integral.

C.2. Region 4b

Both banks are insolvent, but k's insolvency is due to $-k$ and not vice versa. The condition for this is: $A_k + \psi(A_{-k} + B) < D + B \wedge A_{-k} < D$ (mirror of region 2 in the first inequality). Since both banks insolvent, period 2 consumption is the same as in region 4a:

$$c_2(\boldsymbol{\theta}\ |\boldsymbol{\theta} \in \Theta_{4b}) = \frac{A_k + \psi A_{-k}}{(1+\psi)(1-\min[n, x/c])}$$

so that k's integral (the non-constant component) has closed form as before in region 4a (with different boundaries)

$$\Phi(\hat{\theta}_2(A_{-k}),\ \theta + b_4^k\sigma_\nu^2,\ \sigma_\nu) - \Phi(\hat{\theta}_1,\ \theta + b_4^k\sigma_\nu^2,\ \sigma_\nu)$$

The component depending on $-k$ doesn't have closed form because the boundaries of k's integral depend on $-k$. We are left with

$$\Delta_{4b} = \int_{\theta=\theta_i-\epsilon}^{\theta_i+\epsilon} q(\theta) \int_{\theta_{-k}=-\infty}^{\hat{\theta}_1}$$

$$\Big[\Big(\Phi(\hat{\theta}_2(A_{-k}),\ \theta,\ \sigma_\nu) - \Phi(\hat{\theta}_1,\ \theta,\ \sigma_\nu)\Big) -$$

$$\Big(\Phi(\hat{\theta}_2(A_{-k}),\ \theta + b_4^k\sigma_\nu^2,\ \sigma_\nu) - \Phi(\hat{\theta}_1,\ \theta + b_4^k\sigma_\nu^2,\ \sigma_\nu)\Big) \times e^{a_4^k + b_4^k\theta + (b_4^k)^2\sigma_\nu^2/2} \times$$

$$\exp\Big(a_4^{-k} + b_4^{-k}\theta_{-k}\Big)\Big]\phi(\theta_{-k}, \theta, \sigma_\nu)d\theta_{-k}d\theta =$$

$$\int_{\theta=\theta_i-\epsilon}^{\theta_i+\epsilon} q(\theta)\Big\{ \int_{\theta_{-k}=-\frac{x}{1-x}}^{\hat{\theta}_1}$$

$$\Big[\Big(\Phi(\hat{\theta}_2(A_{-k}),\ \theta,\ \sigma_\nu) - \Phi(\hat{\theta}_1,\ \theta,\ \sigma_\nu)\Big) -$$

$$\Big(\Phi(\hat{\theta}_2(A_{-k}),\ \theta + b_4^k\sigma_\nu^2,\ \sigma_\nu) - \Phi(\hat{\theta}_1,\ \theta + b_4^k\sigma_\nu^2,\ \sigma_\nu)\Big) \times e^{a_4^k + b_4^k\theta + (b_4^k)^2\sigma_\nu^2/2} \times$$

$$\exp\Big(a_4^{-k} + b_4^{-k}\theta_{-k}\Big)\Big]\phi(\theta_{-k}, \theta, \sigma_\nu)d\theta_{-k} +$$

$$\Phi(-\frac{x}{1-x}, \theta, \sigma_\nu)\Big[\Big(\Phi(\hat{\theta}_2(0),\ \theta,\ \sigma_\nu) - \Phi(\hat{\theta}_1,\ \theta,\ \sigma_\nu)\Big) -$$

$$e^{a_4^k + b_4^k\theta + (b_4^k)^2\sigma_\nu^2/2} \times \Big(\Phi(\hat{\theta}_2(0),\ \theta + b_4^k\sigma_\nu^2,\ \sigma_\nu) - \Phi(\hat{\theta}_1,\ \theta + b_4^k\sigma_\nu^2,\ \sigma_\nu)\Big)\Big]\Big\}d\theta$$

C.3. Region 4c

Both banks are insolvent, but $-k$'s insolvency is due to k and not vice versa. The condition for this is: $A_k < D \ \wedge \ A_{-k} + \psi(A_k + B) < D + B$ (mirror image of region 3 for the second inequality). Since both banks are insolvent, period 2 consumption is the same as in region 4a:

$$c_2(\boldsymbol{\theta}\ |\boldsymbol{\theta} \in \Theta_{4c}) = \frac{A_k + \psi A_{-k}}{(1+\psi)(1-\min[n, x/c])}$$

so that $-k$'s integral has closed form as before in region 4a (with different boundaries)

$$\Phi(\hat{\theta}_2(A_k),\ \theta + b_4^{-k}\sigma_\nu^2,\ \sigma_\nu) - \Phi(\hat{\theta}_1,\ \theta + b_4^{-k}\sigma_\nu^2,\ \sigma_\nu)$$

The component depending on k doesn't have closed form because the boundaries of $-k$'s integral depend on k. We are left with

$$\Delta_{4c} = \int_{\theta=\theta_i-\epsilon}^{\theta_i+\epsilon} q(\theta) \Big\{ \int_{\theta_k=-\frac{x}{1-x}}^{\hat{\theta}_1}$$

$$\Big[\Big(\Phi(\hat{\theta}_2(A_k),\ \theta,\ \sigma_\nu) - \Phi(\hat{\theta}_1,\ \theta,\ \sigma_\nu) \Big) -$$

$$e^{a_4^{-k} + b_4^{-k}\theta + (b_4^{-k})^2 \sigma_\nu^2/2} \times \Big(\Phi(\hat{\theta}_2(A_k),\ \theta + b_4^{-k}\sigma_\nu^2,\ \sigma_\nu) - \Phi(\hat{\theta}_1,\ \theta + b_4^{-k}\sigma_\nu^2,\ \sigma_\nu) \Big) \times$$

$$\exp\Big(a_4^k + b_4^k \theta_k \Big) \phi(\theta_k, \theta, \sigma_\nu) d\theta_k \Big] +$$

$$\Phi(-\frac{x}{1-x}, \theta, \sigma_\nu) \Big[\Big(\Phi(\hat{\theta}_2(0),\ \theta,\ \sigma_\nu) - \Phi(\hat{\theta}_1,\ \theta,\ \sigma_\nu) \Big) -$$

$$e^{a_4^{-k} + b_4^{-k}\theta + (b_4^{-k})^2 \sigma_\nu^2/2} \times \Big(\Phi(\hat{\theta}_2(0),\ \theta + b_4^{-k}\sigma_\nu^2,\ \sigma_\nu) - \Phi(\hat{\theta}_1,\ \theta + b_4^{-k}\sigma_\nu^2,\ \sigma_\nu) \Big) \Big] \Big\} d\theta$$

D. Simulation: Graphs

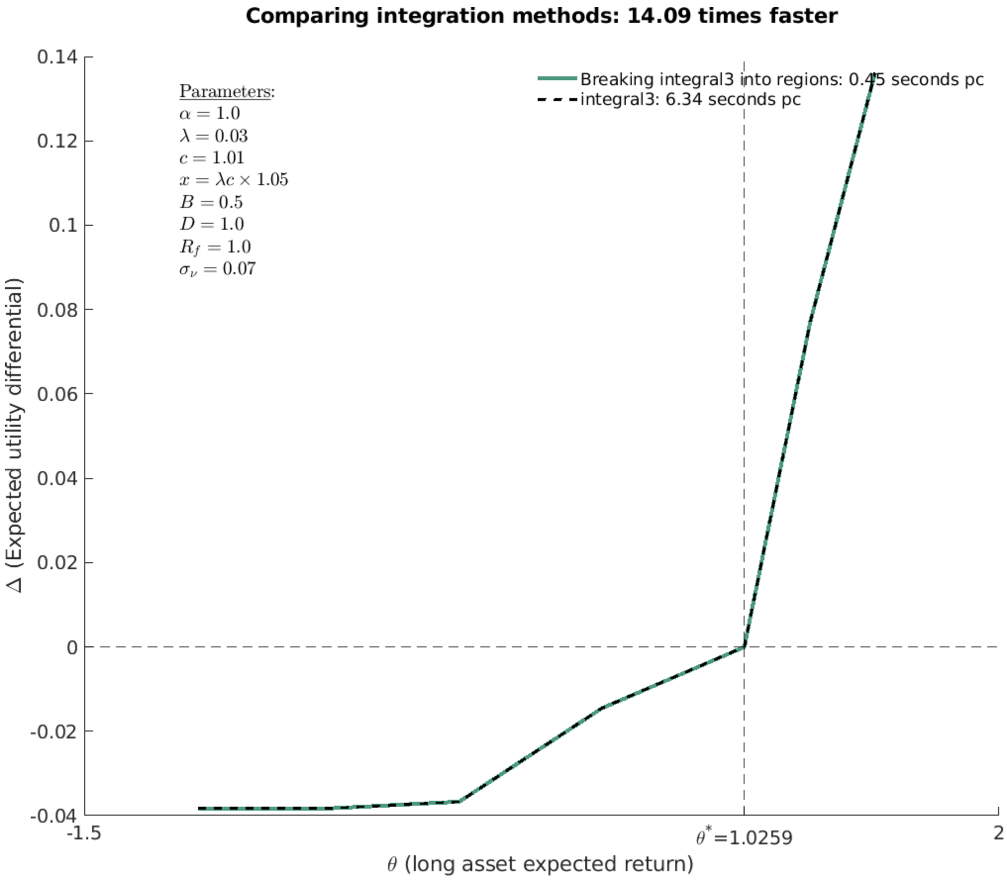


Figure 8. Comparing Integration Methods: Expected Differential Utility aΔ

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