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DOI 10.1007/s11750-007-0010-7

DISCUSSION

Comments on: Static pickup and delivery problems: a classification scheme and survey

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The authors of this survey provide an excellent review on a static class of routing problems called Pickup and Delivery problems. They describe the problems and many of the results for a large collection of pickup and delivery variants that have been considered in the literature. They also propose a classification scheme of the problems with respect to a number of characteristics. Classification schemes of this type are widely used in other fields of operations research such as scheduling problems and queuing problems. I strongly believe that the proposed scheme will be helpful to future researchers looking for problems that remain open.

Variants of the TSP and VRP such as the pickup and delivery problems described in this survey continue to attract the interest of researchers. Several solution methods for analyzing these NP-hard problems have been developed, such as exact solution methods, probabilistic analysis, asymptotic analysis, local search solution methods, empirical analysis of heuristics, and worst-case analysis, the last-mentioned being the main focus of the survey. Pickup and Delivery problems, like other combinatorial optimization problems, have the property that a small variation in the problem definition (like imposing a new constraint, or alternatively relaxing an existing requirement) usually results in a problem with new characteristics, a problem that necessitates the development of new insights. This rich array of problems and solution methods has made the field fascinating to researchers and students. Senior researchers enjoy the experience accumulated in solving such problems, while fresh researchers are sometimes able to produce original ideas. I expect research in this area to continue to be active for at least the next few decades and believe therefore that a classification of the problems such as the one proposed here will certainly be helpful and useful.

This comment refers to the invited paper available at:
http://dx.doi.org/10.1007/s11750-007-0009-0.

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In this discussion I focus on the worst-case analysis results which are described extensively in the survey for a large number of TSP and VRP variants that deal with pickup and delivery operations. The interest in polynomial-time approximation algorithms for NP-hard problems is as old as the development of complexity theory in the early 1970s and developing such an algorithm that is guaranteed to produce a solution within $100\alpha\%$ of the minimum cost solution for constant α, especially for small values of α, is certainly comforting in view of the commonly accepted belief regarding the hardness and insolvability of the problem. However, it is important to note that, in general, the approximation algorithms which are designed to reach a low worst-case bound are not the ones that are widely used by practitioners when solving real-life problems. The reason for their limited use in practice is that in designing an approximation algorithm whose worst-case bound is low, one must consider the pathological instances, i.e., instances in which the proposed scheme is expected to behave in the worst possible way. Such instances are rare in reality, and from a practical point of view one needs an algorithm that is designed to work well on an average-type instance.

Much of the effort that has been devoted during the last three decades to research on worst-case analysis of the TSP and its variants has clearly not been for practical reasons. However, it is evident today that the development and progress of the field is different from the development of research on worst-case analysis of some other NP-hard problems, such as the bin-packing problem or some of the scheduling problems. In the following I will briefly elaborate on this last assertion. The research stream on worst-case analysis of NP-hard combinatorial optimization problems started in the 1970s, along with the progress then being made in understanding complexity theory: it was realized at that time that as the TSP belonged to the class of NP-hard problems, a polynomial algorithm for solving the TSP to optimality was not probable. The next important negative result was obtained by Sahni and Gonzalez (1976) who proved that unless $P = NP$ there can be no efficient approximation algorithm for the TSP that achieves some bounded approximation ratio. Fortunately, in symmetric metrics obeying the triangle inequality, polynomial-time approximation algorithms with bounded worst-case ratios have been developed for the TSP and many of its variants, as described in this extensive survey. The best known worst-case bound for the TSP was proposed 30 years ago(!) in the seminal paper by Christofides (1976). In this paper Christofides designed an algorithm of complexity cubic in the number of vertices, that is, guaranteed to generate a solution at most 50% above the optimal cost solution. Later, Cornuéjols and Nemhauser (1978) constructed an instance of the TSP on which Christofides' algorithm produces a solution that is exactly 150% of the optimal cost, meaning that the worst-case bound $\alpha = 1.5$ is tight. Since then no breakthrough result in this area has been obtained, i.e., no approximation algorithm with a better worst-case bound for the TSP has been found, despite the enormous effort invested in this endeavor in recent decades by many researchers all over the world. Though researchers have introduced and investigated new variants of the TSP, as for example pickup and delivery problems, the worst-case analysis of these variants when defined on general symmetric graphs that obey the triangle inequality is strongly dependent on the 1.5 bound obtained for the basic TSP, as the approximation algorithms usually include a component that necessitates the connection of a number of points by a closed tour. This fact is striking, especially in view of the progress that has been made on worst-case analysis of approximation algorithms proposed for bin-packing and scheduling problems. In these problems, the variants are more independent in the sense that their worst-case analysis is not interrelated with a single basic problem.

Today, it seems that researchers working on worst-case analysis of the TSP and its pickup and delivery variants conceive of the current state-of-the-art in this area as an equilibrium state, an equilibrium state reached over 30 years ago. In other words, the maximum gap of 50% between the quality of the solution obtained by the Christofides algorithm and the optimal solution for the basic TSP have remained intact for the last three decades. A better worst-case bound for the TSP would certainly improve the worst-case bounds for many of the variants of the TSP and VRP on general graphs. I look forward to the spark of genius igniting research of worst-case analysis of routing problems to produce a breakthrough result, either in the form of a new approximation algorithm for the basic TSP having a worst-case bound lower than 1.5, or an understanding of the inherent difficulty in doing so.

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